Exercise: Formulate integer programming problems

1 Linear programming problem properties

Find necessary and sufficient conditions for the numbers s and t to make the linear programming problem

$$\begin{aligned} &\text{maximize} && x_1 + x_2 \\ &\text{subject to} && s \cdot x_1 + t \cdot x_2 \leq 1 \\ & && x_1, x_2 \geq 0 \end{aligned}$$

(i) has an optimal solution,

(ii) be infeasible,

(iii) be unbounded.

2 LP formulation

A meat packing plant produces 480 hams, 400 pork bellies, and 230 picnic hams every day; each of these products can be sold either fresh or smoked. The total number of hams, bellies, and picnics that can be smoked during a normal working day is 420; in addition, up to 250 products can be smoked on overtime at a higher cost. The *net* profit are as follows.

	Fresh	Smoked on regular time	Smoked on overtime
Hams	\$8	\$14	\$11
Bellies	\$4	\$12	\$7
Picnics	\$4	\$13	\$9

The objective is to find the schedule that maximizes the total net profit. Formulate as an LP problem.

3 Variation of the knapsack problem

Suppose that you are interested in choosing a set of investments $\{1, \cdots, 7\}$ using $0-1$ variables. Model the following constraints:

(i) You must choose at least one of them.

(ii) You cannot invest in all of them.

4 Simplex method

Consider the linear programming problem:

$$\begin{array}{lrcrl} \text{maximize} & 40x & + & 30y & \\ \text{subject to} & x & + & 2y & \leq 16 \\ & x & + & y & \leq 9 \\ & 3x & + & 2y & \leq 24 \\ & & & x, y & \geq 0 \end{array}$$

Recall that during the lecture, we started at the feasible solution $(x, y) = (0, 0)$, and then improved the objective value by fixing $x = 0$ and increasing y. Please run the simplex method by fixing $y = 0$ and increasing x.