Exercise Solution: P and NP

Prove their NP membership for the following problems (or their decision version).

1. **Partition.** Given a set S of n integers $a_1, a_2, \cdots, a_n$, is there a way to partition S into two subsets such that the sums in each subset are the same?

2. **Conjunctive normal form satisfiability.** A Boolean formula is in *Conjunctive Normal Form (CNF)* if it is a conjunction of clauses, where a clause is a disjunction of literals. Given a CNF formula, is it satisfiable?

3. **Minimum vertex cover.** Given graph $G = (V, E)$, a *vertex cover* is a subset of vertices $C \subseteq V$ such that for all edge $(u, v) \in E$, u or v is in C. The minimum vertex cover problem asks about finding the minimum vertex cover in the given graph.

4. **Maximum independent set.** An *independent set* is a set of vertices where each pair of vertices is not adjacent to each other. The *maximum independent set problem* is: given a graph G, what is the size of the maximum independent set in G?

5. **Bin packing.** Given n items, where each item i with size $s_i \in (0, 1]$, the goal is packing the items into the minimum number of capacity-1 bins.

6. **Knapsack.** There are n items, each with positive integral *weight* w_j $(j = 1, \cdots, n)$ and positive integral *value* c_j $(j = 1, \cdots, n)$ and an integer b. The question is to find a subset of the items with total weight at most b and the maximal total value.