Exercise Solution: NP-Completeness

1. Let DOUBLE-SAT $= \{\langle \phi \rangle \mid \phi$ has at least two satisfying assignments$\}$.

   Show that DOUBLE-SAT is NP-complete.

2. Show that PARTITION is NP-complete. (Hint: By reduction from SUBSETSUM.)

3. The language 2WAYPARTITION is defined as $\{\langle S, t\rangle \mid S = \{x_1, x_2, \cdots, x_n\}$ where exists a subset $T \subset S$ such that $\sum_{y_i \in T} y_i = \sum_{y_i \in S \setminus T} y_i$ and $|T| \leq t$ and $|S \setminus T| \leq t\}$. Show that 2WAYPARTITION is NP-hard.

4. Given a graph $G = (V, E)$, an *independent set* is a subset U of vertices in V such that there is no edge between any two vertices in U. In the *Maximum Independent Set* problem, we aim to find the maximum independent set in the given graph.

   (a) Give the decision version of the Maximum Independent Set, INDEPSET

   (b) Show that the decision version of the Maximum Independent Set is NP-complete.

5. Given a graph $G = (V, E)$, a *vertex cover* is a subset C of vertices in V such that for any edge $(u, v) \in E$, $\{u, v\} \cap U \geq 1$. In the *Minimum Vertex Cover* problem, we aim at finding the minimum vertex cover in the given graph.

   (a) Give the decision version of the Minimum Vertex Cover, VC

   (b) Show that the decision version of the Minimum Vertex Cover, VC, is NP-complete.

6. The weighted vertex cover problem is defined as follows. Given a graph $G = (V, E)$, where each vertex $v \in V$ has a positive weight w_v, find a subset of V with minimum total weight such that this subset forms a vertex cover of G. Show that the weighted vertex cover problem is NP-complete.

   (Note that you need to provide formal proof; saying ”since its special case vertex cover is NP-hard, it is NP-hard” is insufficient.)