Exercise: Different formulations and Branch-and-Bound

1 Minimum Spanning Tree

Recall the minimum spanning tree problem mentioned in the lecture. There is another way to describe the connectivity of the minimum spanning tree. That is, there should be exactly $|V|-1$ edges selected. Use the new description to formulate the minimum spanning tree problem.

We define variable x_{uv} for every edge (u, v) such that $x_{uv} = 1$ if edge (u, v) is selected, and $x_{uv} = 0$ otherwise.

$$\begin{array}{llll} \text{minimize} & \displaystyle\sum_{(u,v)\in E} c_{uv}x_{uv} & & \\ \text{subject to} & \displaystyle\sum_{(u,v)\in E} x_{uv} = & |V|-1 & \\ & \displaystyle\sum_{(u,v):(u,v)\in E, u\in S, v\in S} x_{uv} \leq & |S|-1 & \text{for any } S \subseteq V \\ & x_{uv} \in & \{0,1\} & \text{for } (u,v)\in E \end{array}$$

2 Lot-Sizing

Recall the lot-sizing problem mentioned in the lecture. There is another way to describe the feasibility of a schedule: On any day t, the total production so far is enough for the total demand so far. Then, you can replace the variable s_t by $\sum_{i=1}^{t} x_t - \sum_{i=1}^{t} d_t$. Use this description to formulate the lot-sizing problem.

$$\begin{array}{llll} \text{minimize} & \displaystyle\sum_{t=1}^{n}(p_t + h_t + h_{t+1} + \cdots + h_n)x_t + \sum_{t=1}^{n} f_t y_t - \sum_{t=1}^{n} h_t(\sum_{i=1}^{t} d_i) & & \\ \text{subject to} & \displaystyle\sum_{i=1}^{t} x_t - \sum_{i=1}^{t} d_t \geq & 0 & \text{for } t = 1, 2, \cdots, n \\ & x_t \geq & 0 & \text{for all } v \in V \end{array}$$

3 Bin Packing Problem

In the (offline) BinPacking problem, there are infinite size-1 bins available for optimally packing n items, where item i has a size of $s_i \in (0, 1]$. Write an ILP formulation for the BinPacking problem.

Since an item needs at most one n bin, we can consider the case where there are only n bins. We introduce indicator variable $y_j \in \{0, 1\}$ for each bin j. Let y_j be 1 is bin j is opened, and $y_j = 0$

otherwise. Let $x_{ij} = 1$ indicate if item i is assigned to bin j, and $x_{ij} = 0$ otherwise.

$$
\begin{aligned}
\text{minimize} \quad & \sum_{j=1}^{n} y_j & \\
\text{subject to} \quad & \sum_{i=1}^{n} x_{ij} \cdot s_i \leq y_j & \text{for } j \in [1, n] \\
& \sum_{j=1}^{n} x_{ij} = 1 & \text{for all } i \in [1, n] \\
& x_{ij} \in \{0, 1\} & \text{for all } i, j \in [1, n] \\
& y_j \in \{0, 1\} & \text{for all } i, j \in [1, n]
\end{aligned}
$$