Exercise 8: Problem Lower Bounds

1 Ever Given

Recall the Ever Given problem, where waiting until the canal is available takes F units of time, and going around Africa via the Cape of Good Hope takes S. Prove that for this problem, there is no deterministic online algorithm better than $(2 - \frac{F}{S})$-competitive.

2 Two-store Ski-Rental

Consider that there are two stores in the Ski-rental problem, 1 and 2, where you can buy or rent a pair of skis. Let r_1 and B_1 be the renting and buying prices from Store 1, respectively. For Store 2, r_2 and B_2 are defined symmetrically. Note that there is no specific relation between r_1 and r_2 or B_1 and B_2, except that B_1 and B_2 are both larger than $\max\{r_1, r_2\}$. Let $r_{\min} = \min\{r_1, r_2\}$ and $B_{\min} = \min\{B_1, B_2\}$. Show that the problem lower bound is at least $2 - \frac{r_{\min}}{B_{\min}}$.

(Hint: You can first show that the algorithms that always rent from the store with the lower renting price and buy from the store with a lower buying price must have a better ratio. Then, you focus on the algorithms in this family and find adversarial instances for any of them.)

3 Online Load Balancing on Two Machines

Recall the online load balancing problem, where there are m machines. A sequence of n jobs arrives where each job J_i has a processing load of ℓ_i. The goal is to assign the jobs so that the highest load of the machines is minimized. Show that when $m = 2$, no online algorithm can be better than 1.5-competitive.

4 Potential function of Ski-Rental

Recall the Ski-Rental problem and the algorithm that buys the ski on the B-th skiing day. Use the potential function method to show that the algorithm is 2-competitive.

(Hint: In this problem, the parameters observable from outside are: if the algorithm/optimal have a pair of ski, and how many skiing days are there so far.)