Beyond the Worst Case Machine-Learned Advice

Algorithms for Decision Support

Outline

- How to "trust" an advice when there is no guarantee from the advice
 - Searching
 - Ski-rental problem
 - 3 algorithms
 - Bin packing

input)

Deal with Uncertainty

• Online algorithms deal with optimization under uncertainty (about future

- input)
 - learning?

Online algorithms deal with optimization under uncertainty (about future)

• How if the future information can be predicted or learned by machine-

- input)
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 - Example: weather forecast

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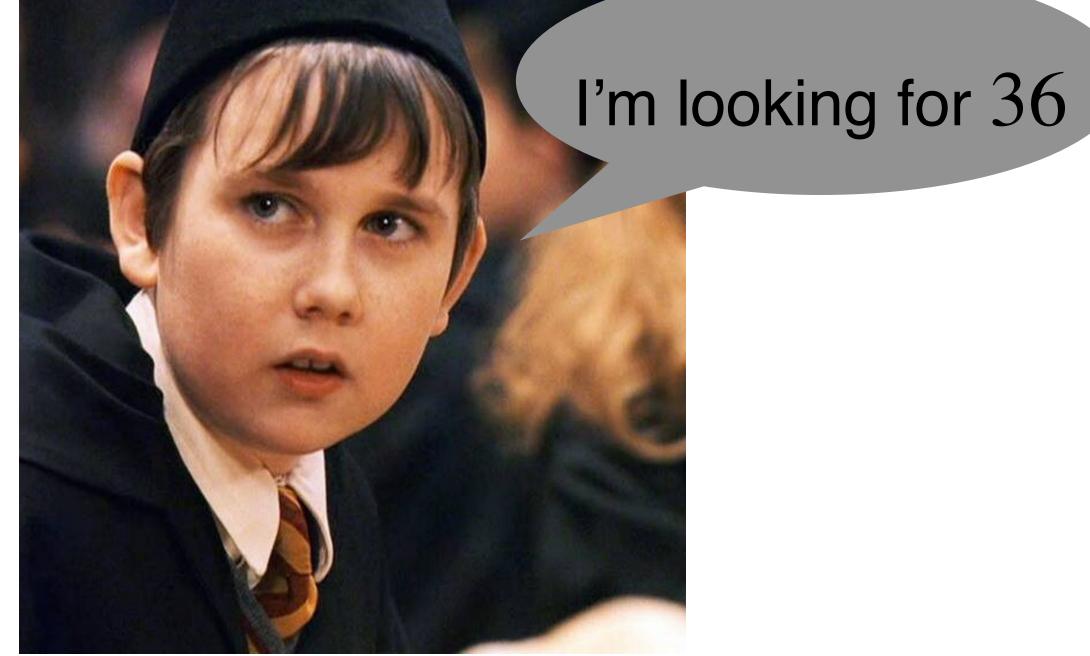
These predictions or learned information may not be 100% correct

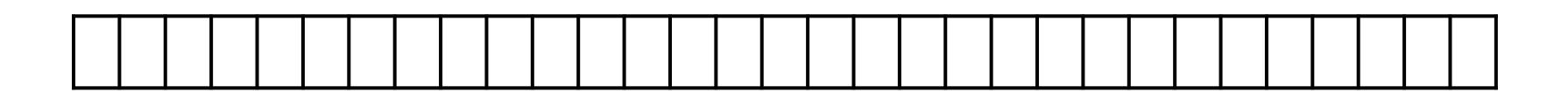
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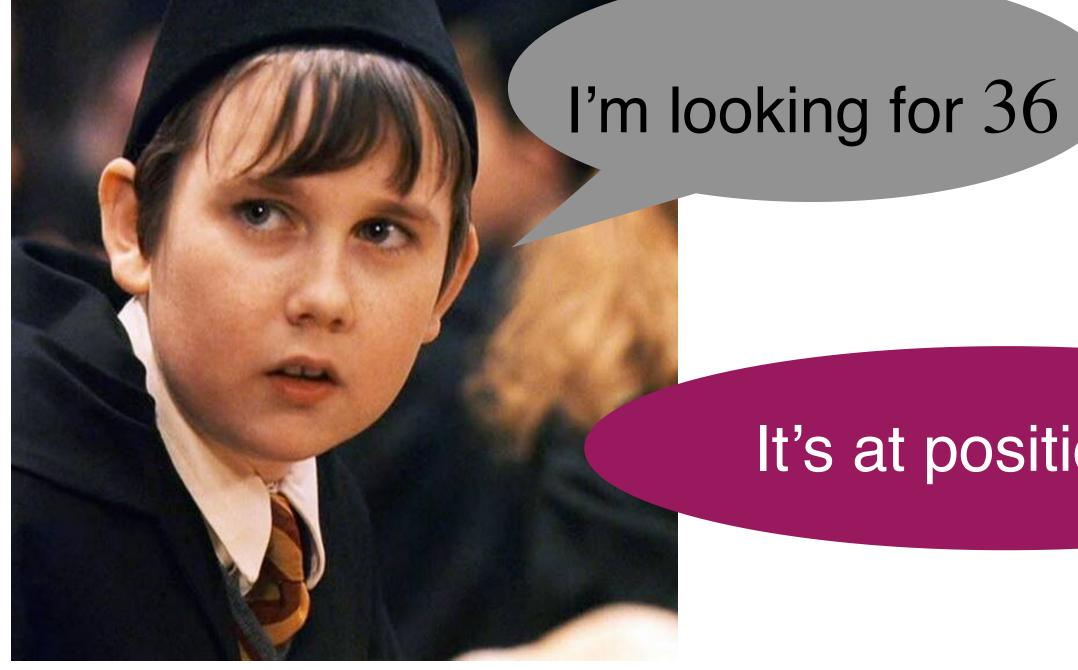
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 - How can we design online algorithms with this kind of (maybe) untrusted advice?

 Goal: Given an ordered sequence and a requested value, find the requested value in minimum number of steps

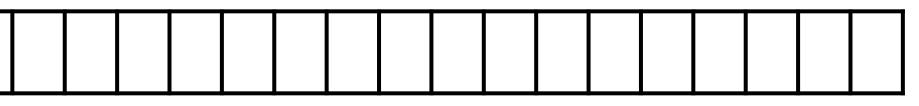


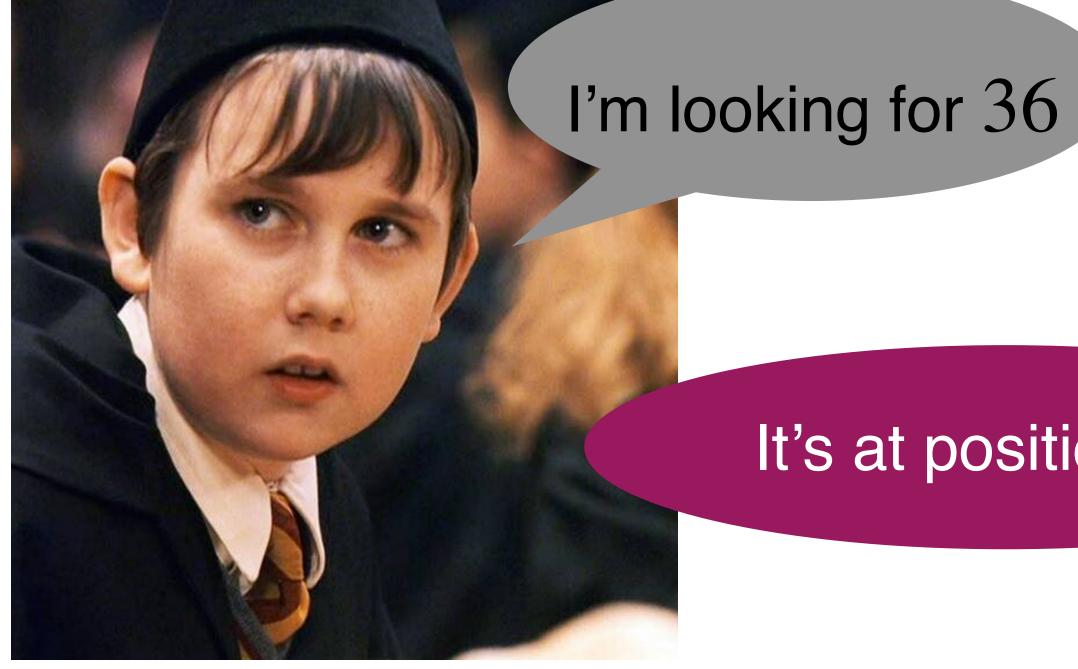




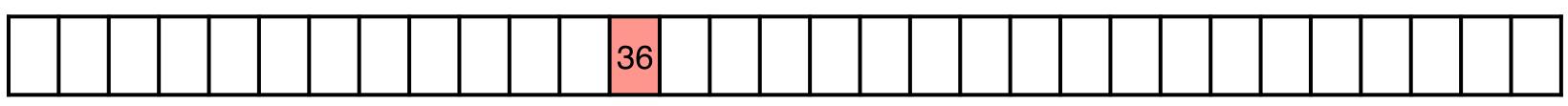
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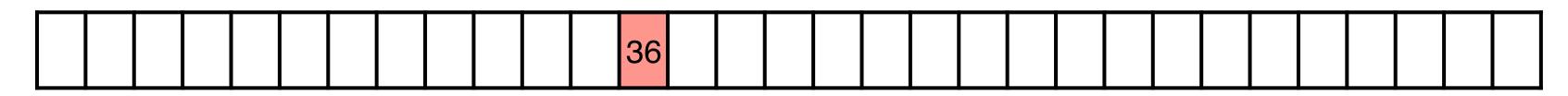
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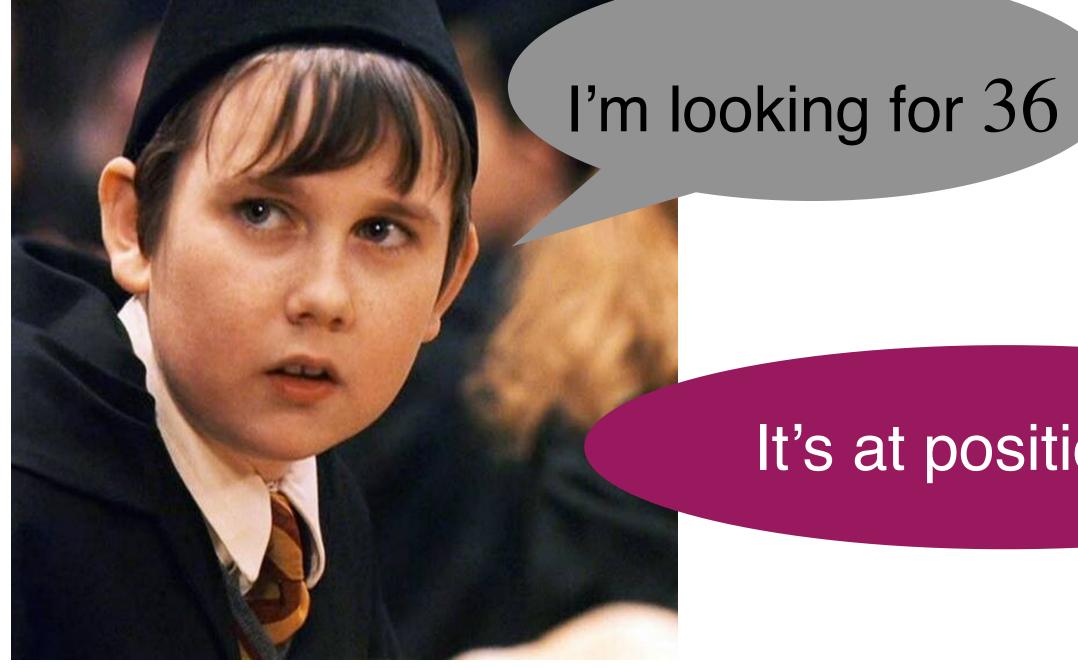


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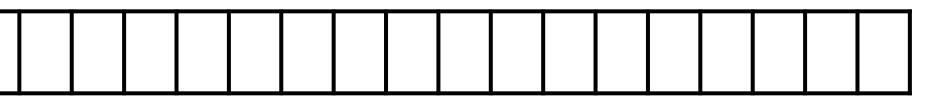


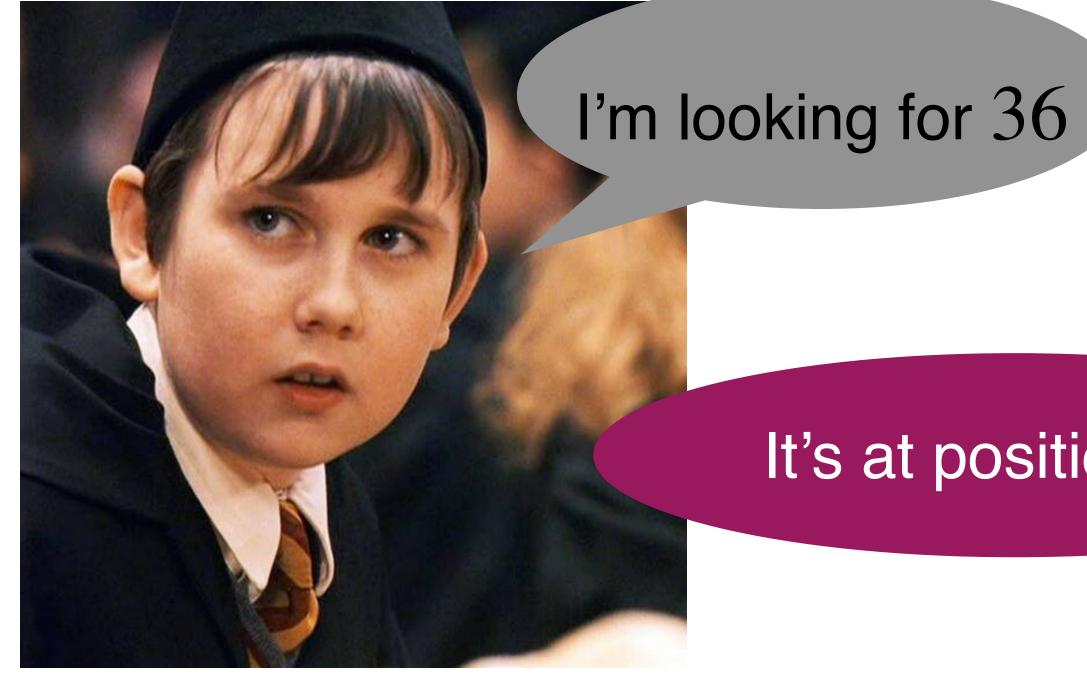
Cost = 1



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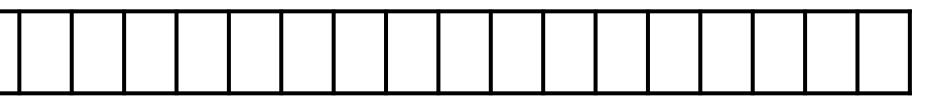


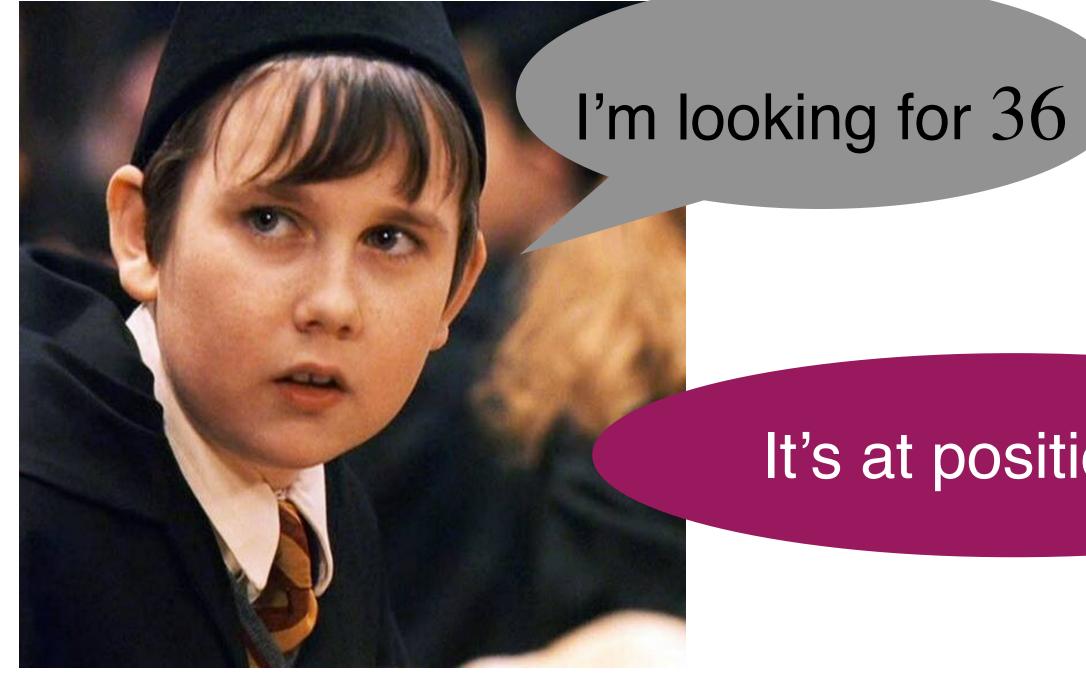


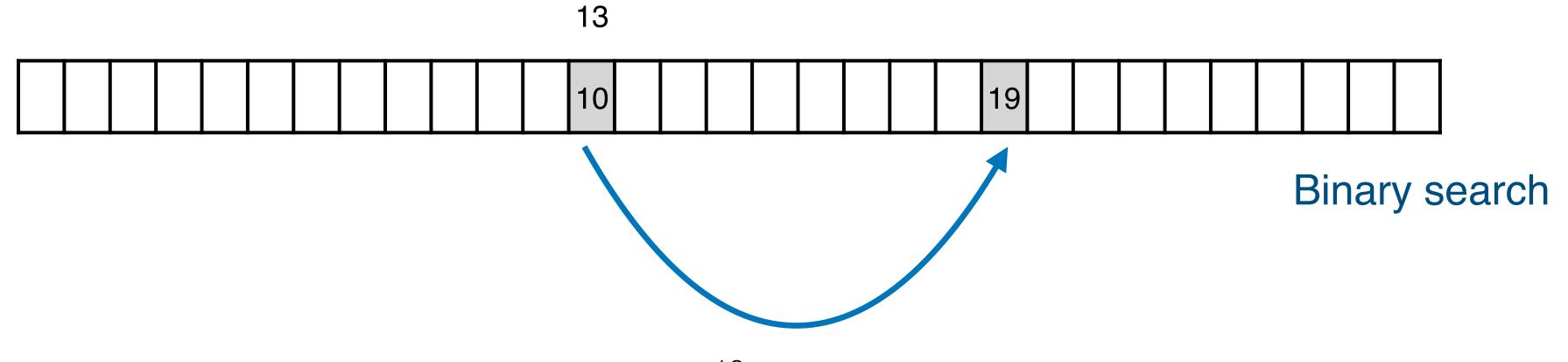
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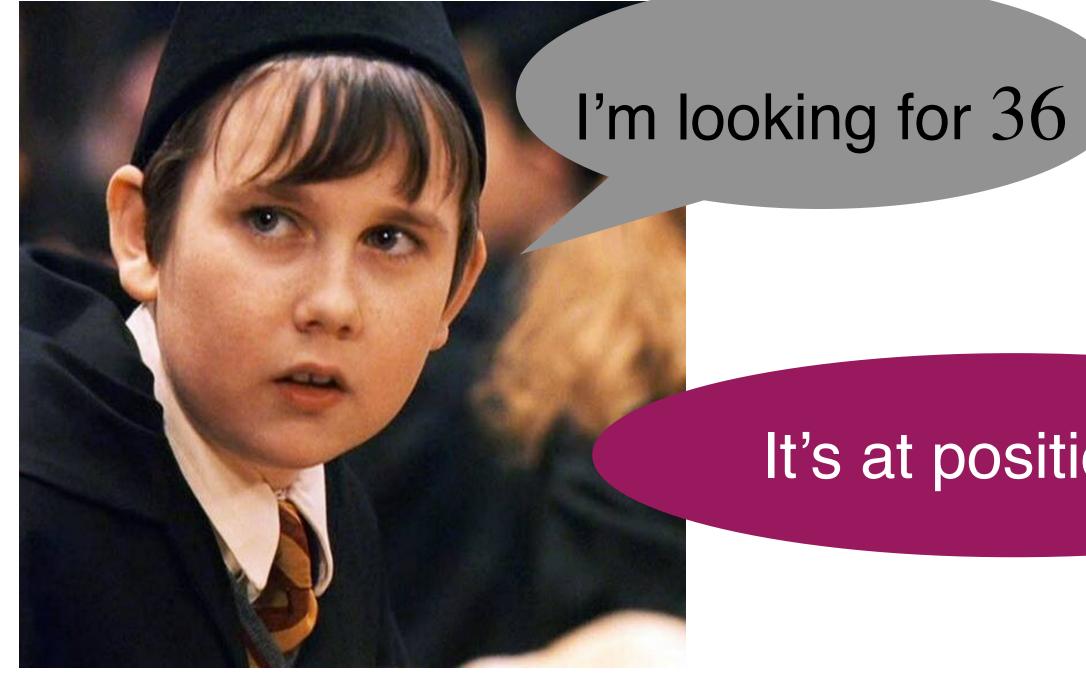


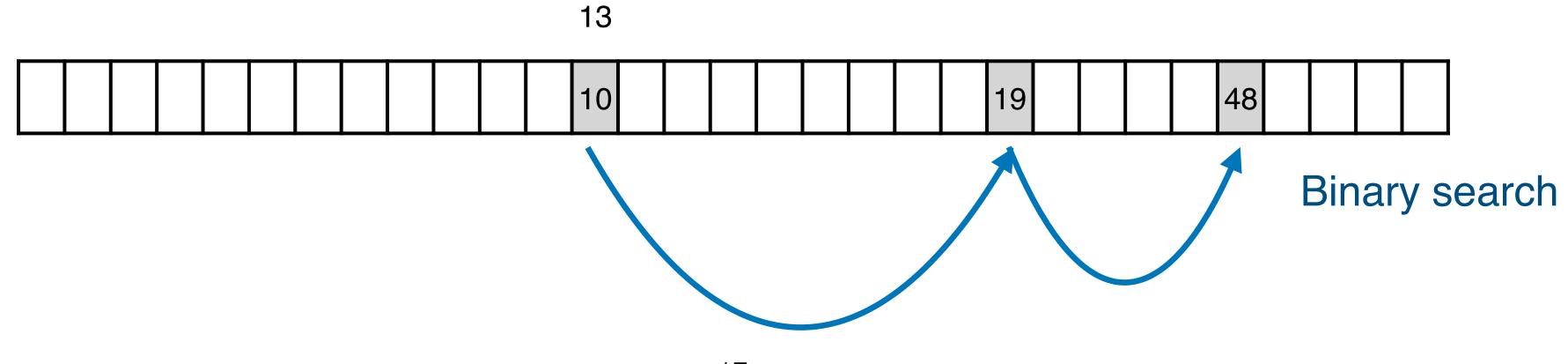




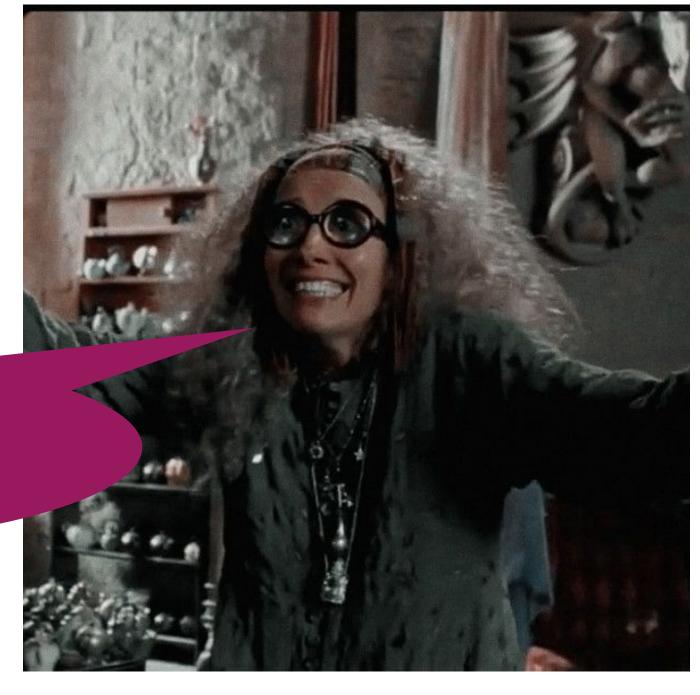


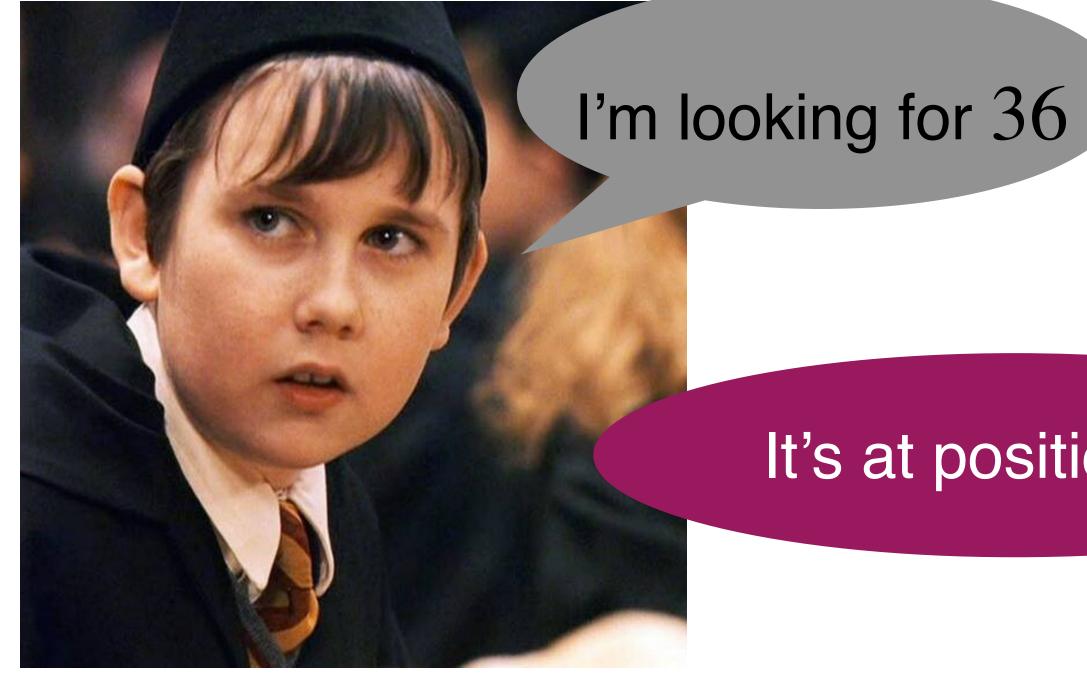


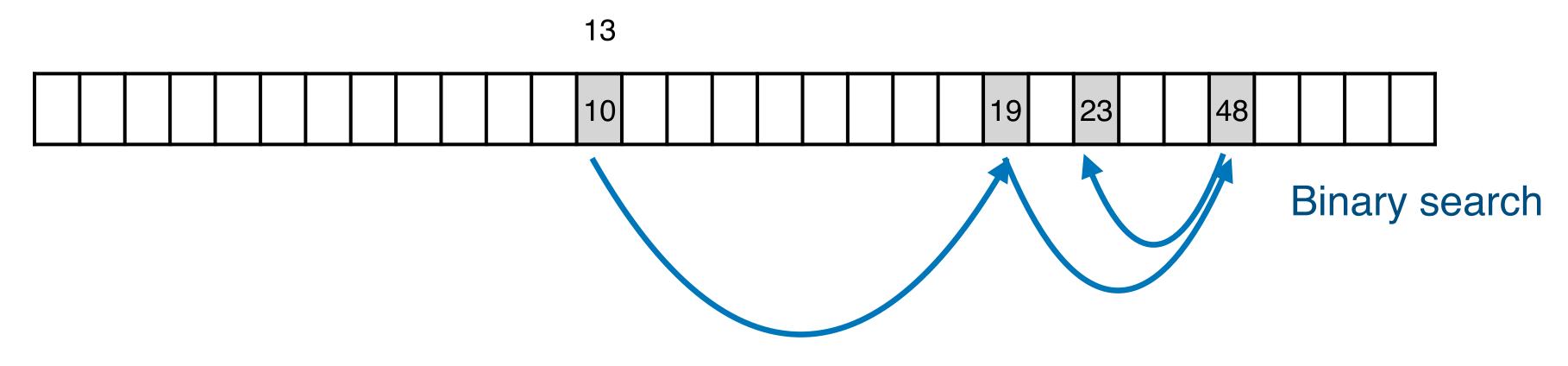






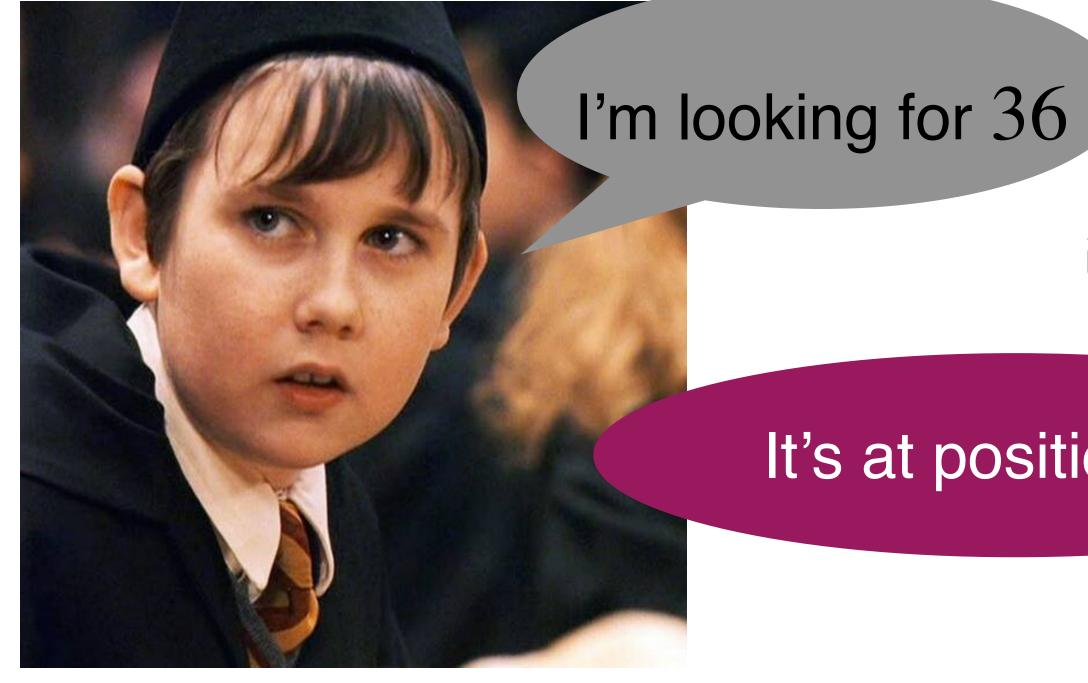


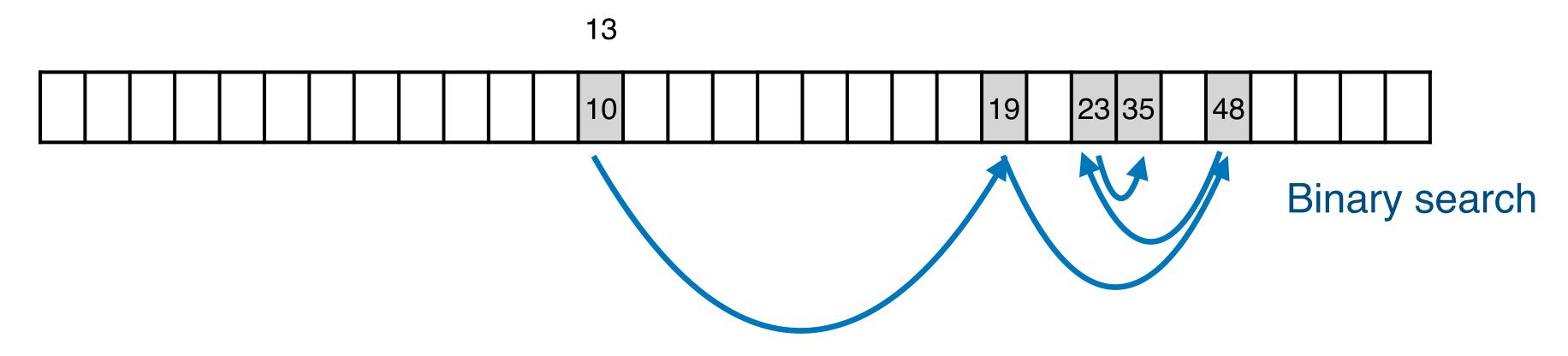






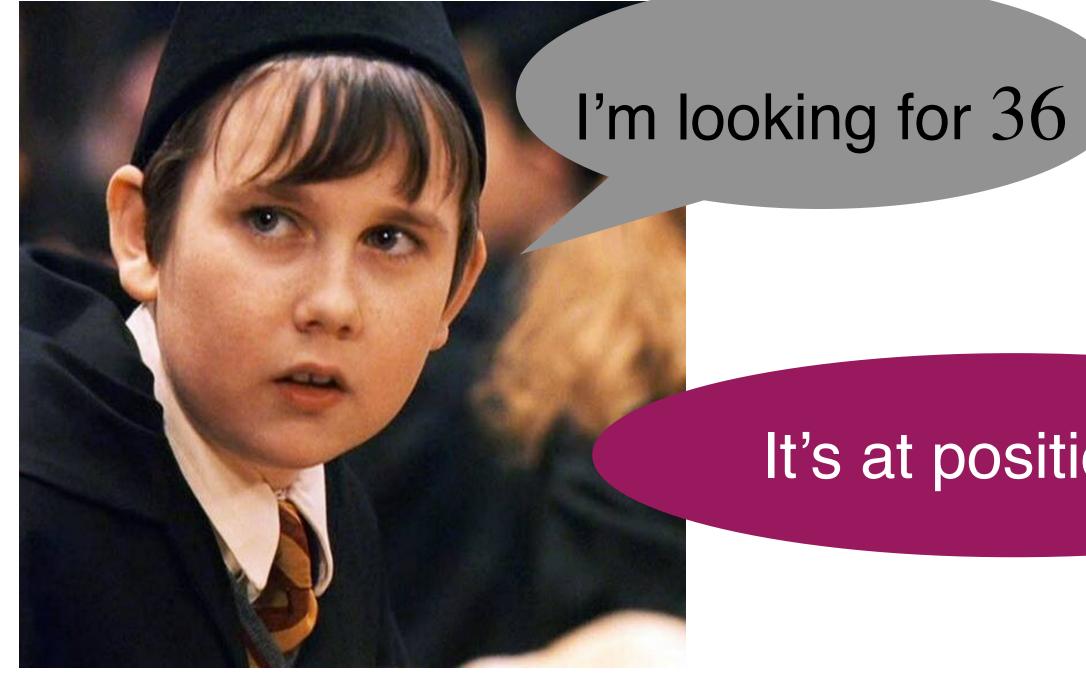


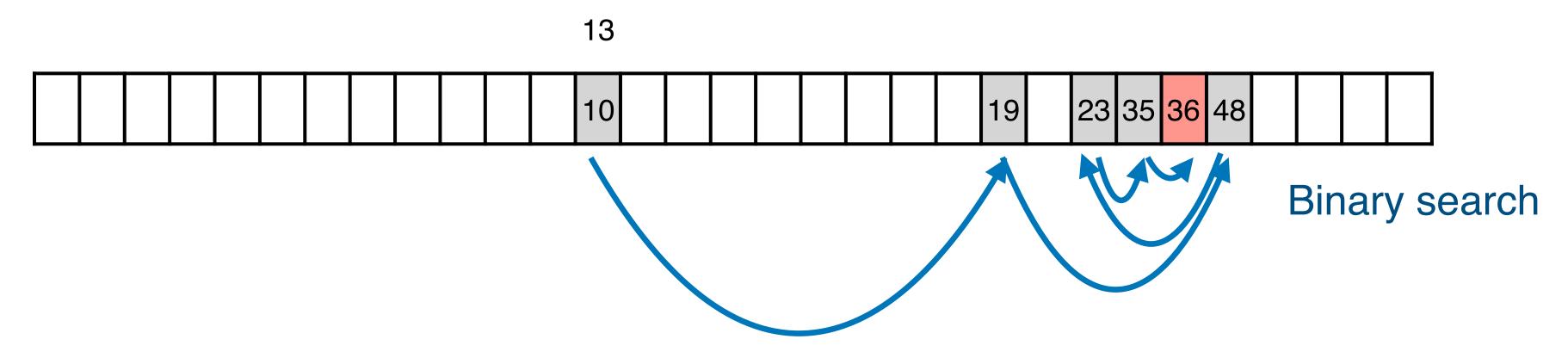




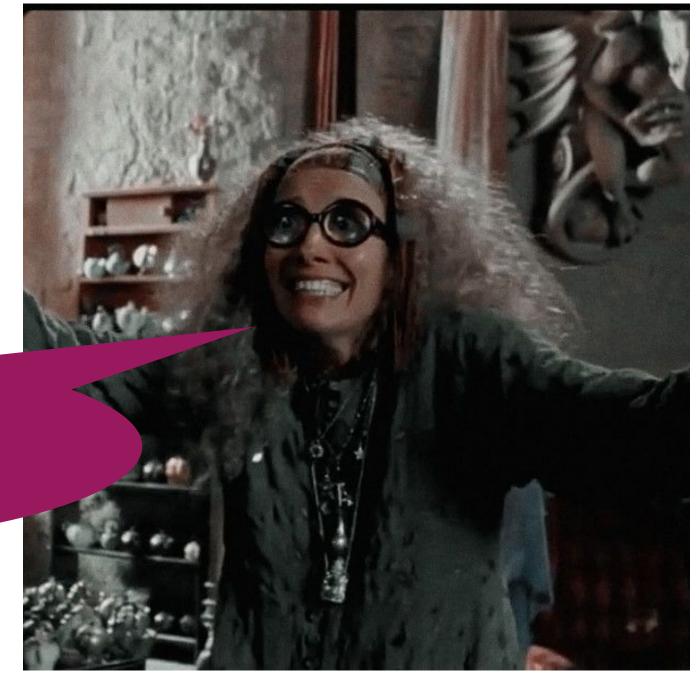


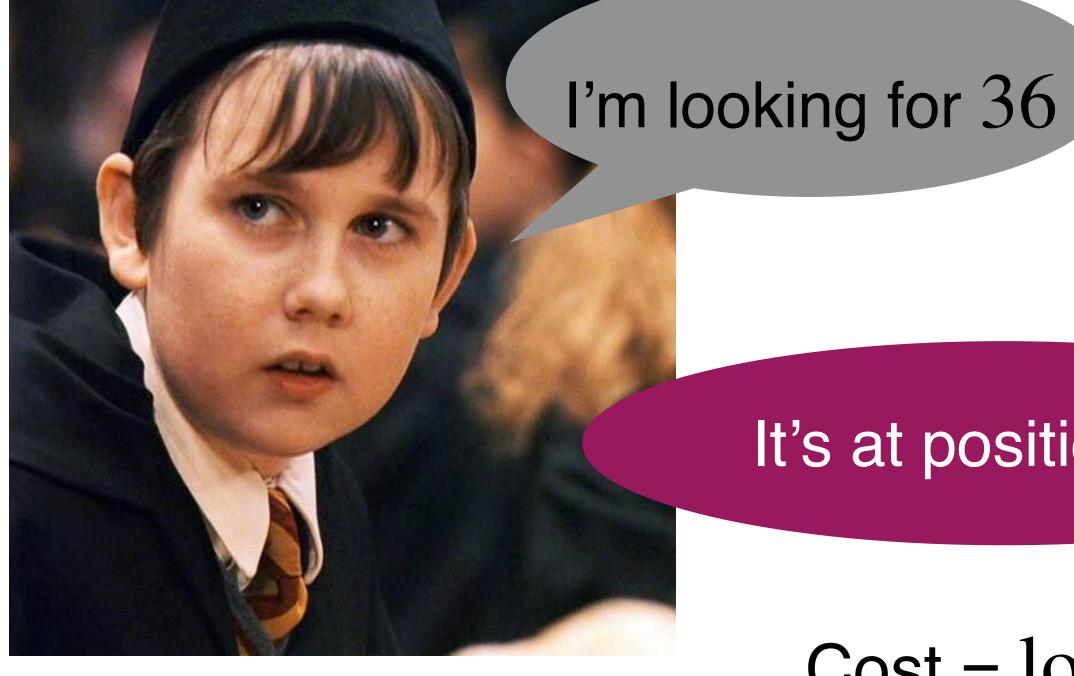




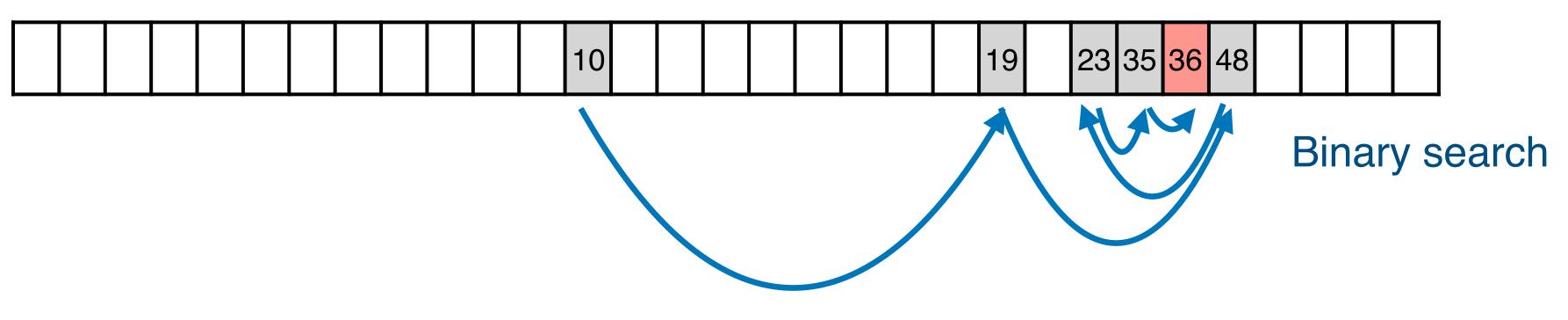








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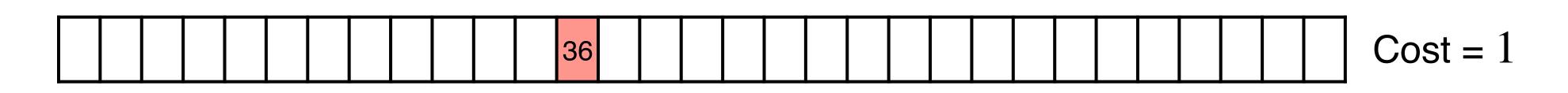


It's at position 13!

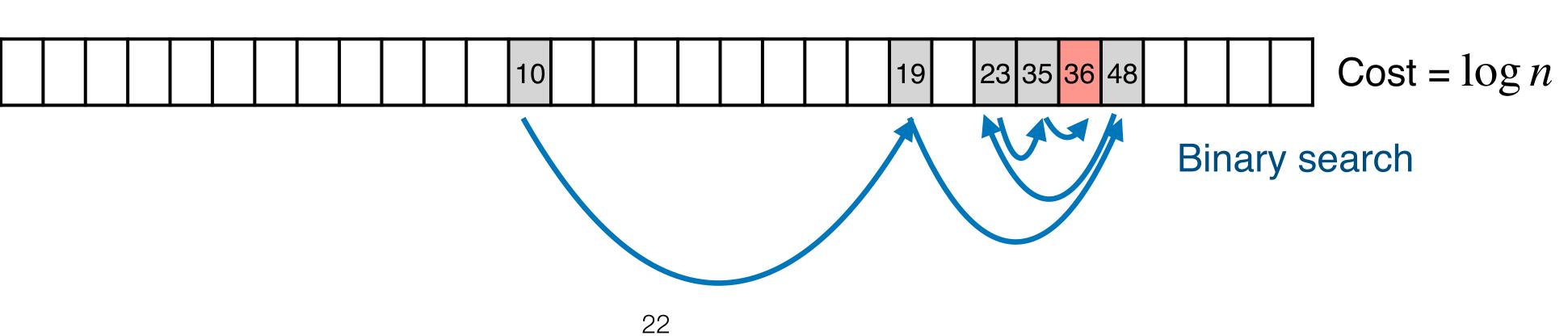
Cost = log n

Consistency and Robustness

- We look for algorithms that
 - Are consistent: given more accurate predictions, the online algorithm should perform close to the optimal offline algorithm



• Are robust: if the prediction is wrong, the online algorithm performance should be close to the online algorithm without predictions





Use predictions to improve the algorithm's performance

(But the prediction can be wrong)

Idea

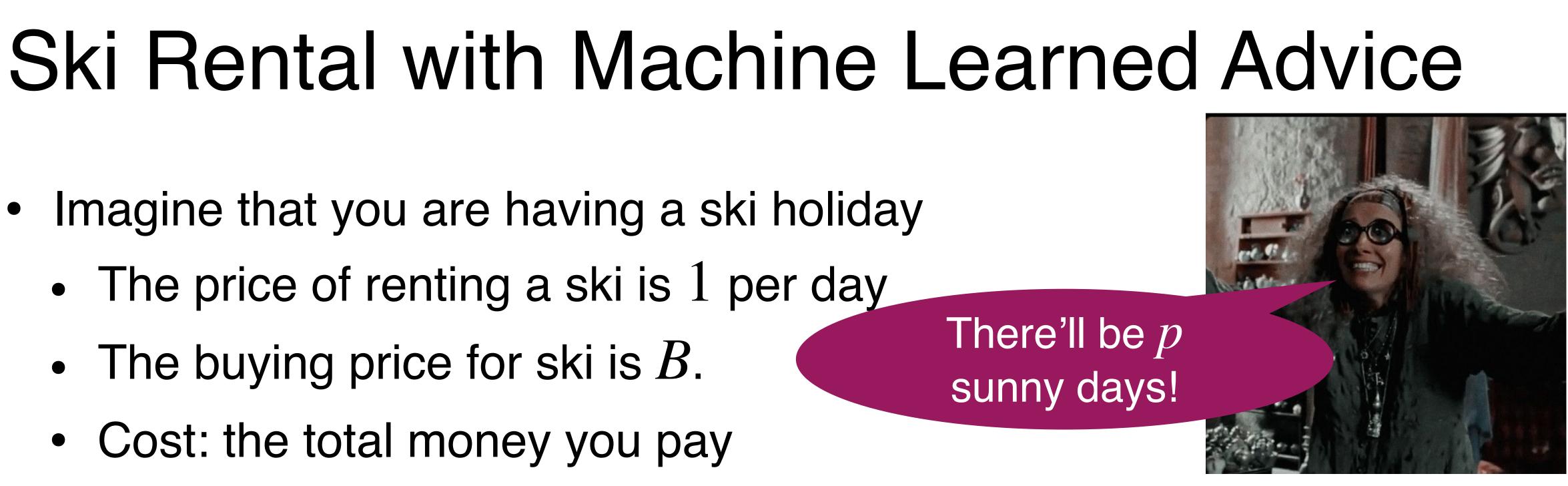
Ski Rental

- Imagine that you are having a ski holiday
 - The price of renting a ski is 1 per day
 - The buying price for ski is *B*.
 - Cost: the total money you pay

you buy the ski or rent it?

Suppose you want to spend money as little as possible. Should

- Imagine that you are having a ski holiday
 - The price of renting a ski is 1 per day
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• p: machine-learned prediction on the number of skiing days • Suppose you want to spend money as little as possible. Should

SKI-Rental with prediction (p)If $p \geq B$ Buy the ski on the first day else (p < B) Keep renting for all skiing days

- // p: prediction on the number of skiing days

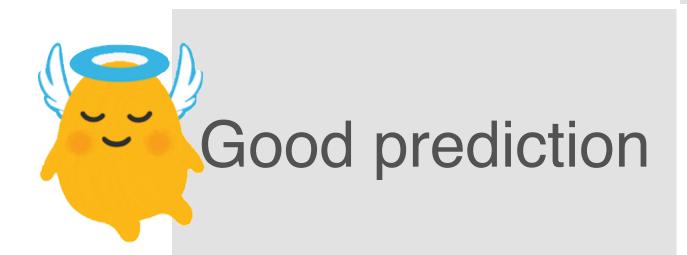
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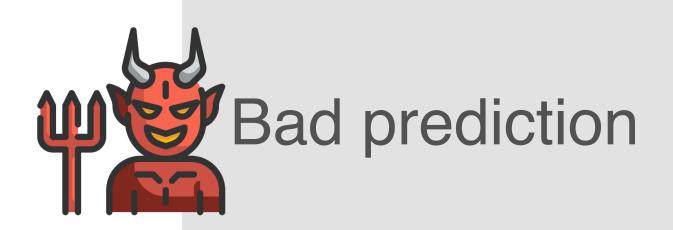
Truth: $d \geq B$ (OPT buy)

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- *d*: Actual number of skiing days Truth: d < B (OPT rent)

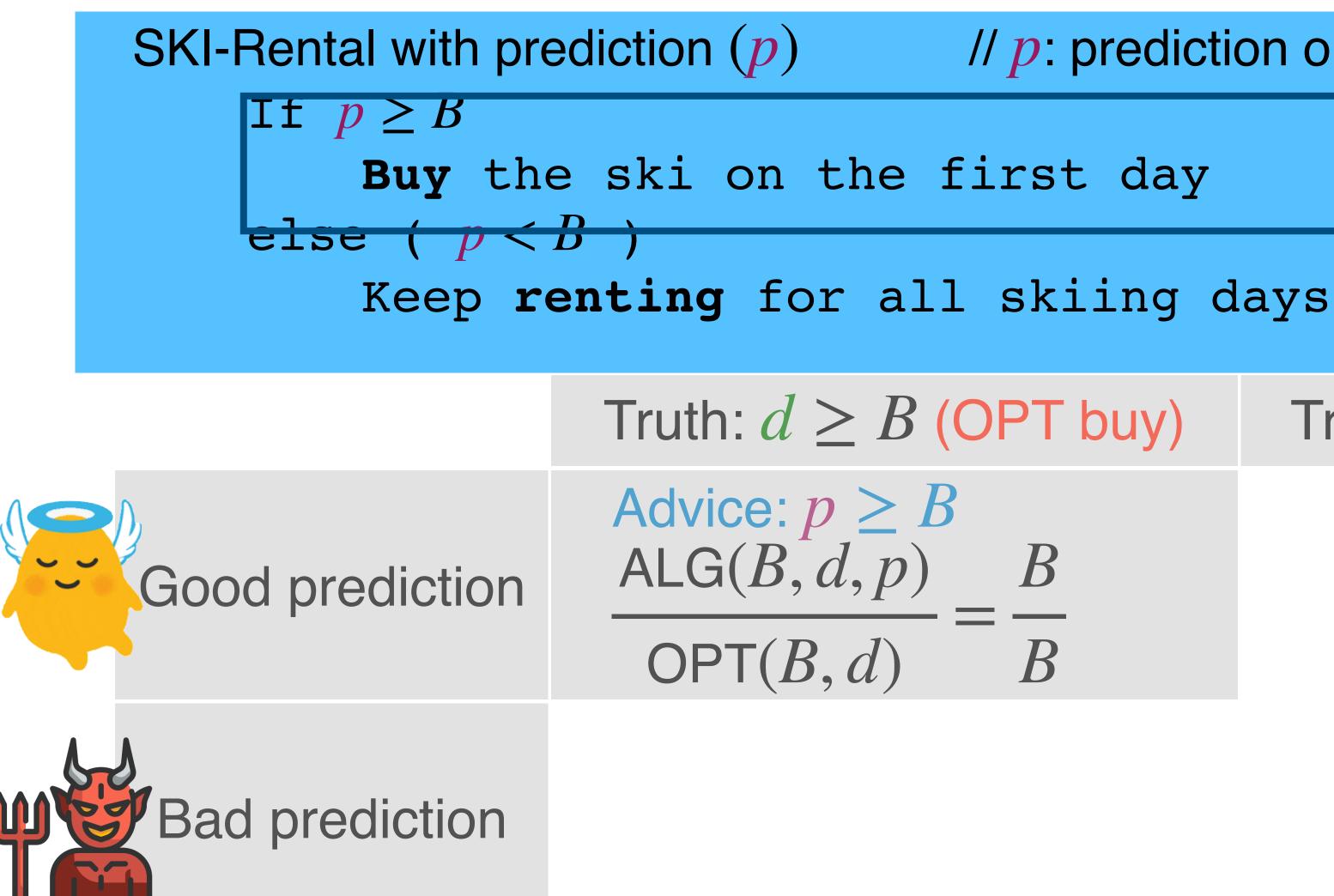
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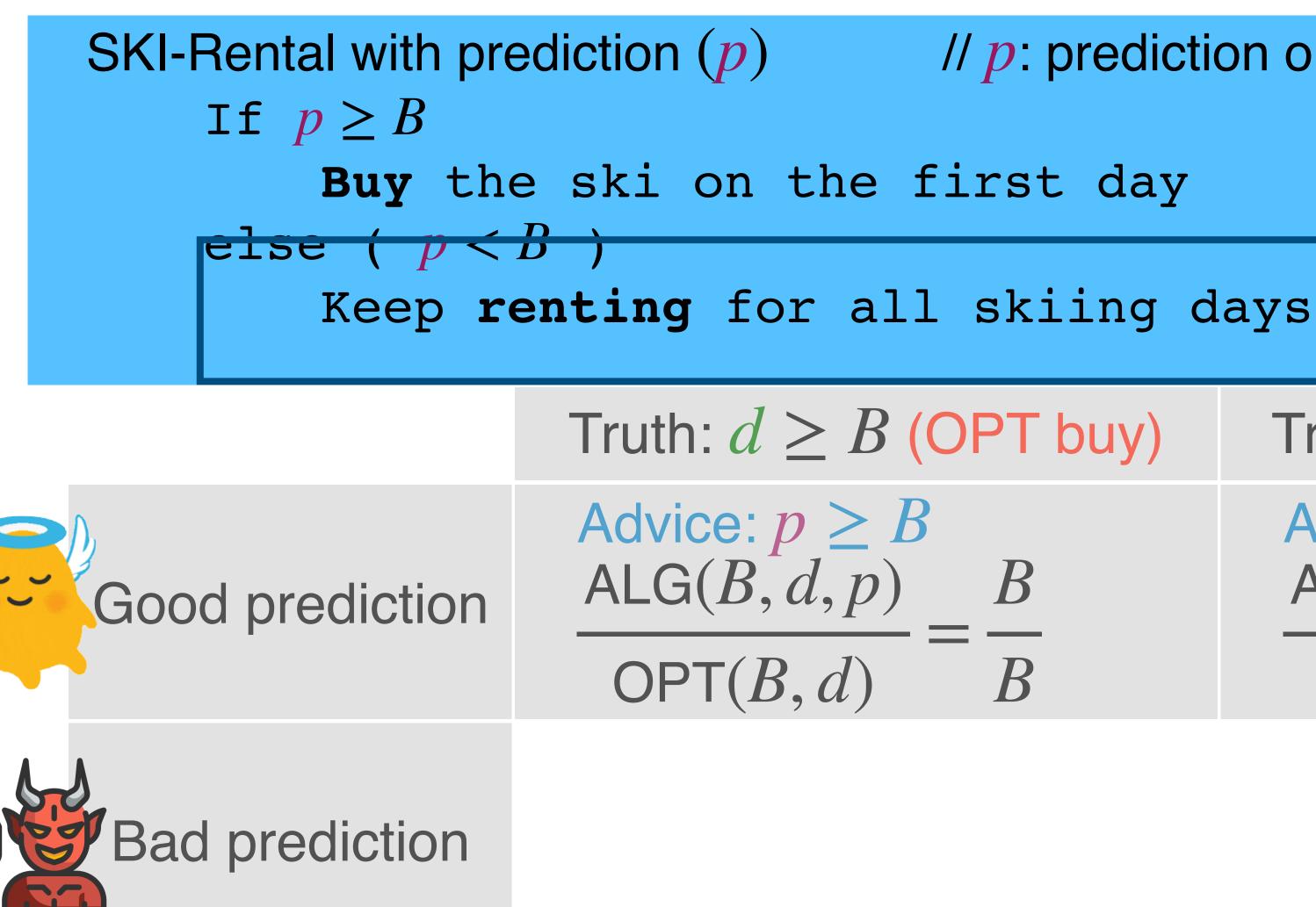


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// p: prediction on the number of skiing days

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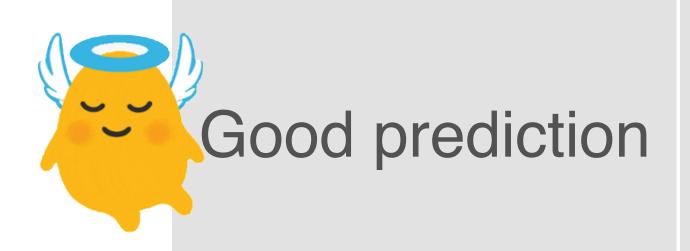


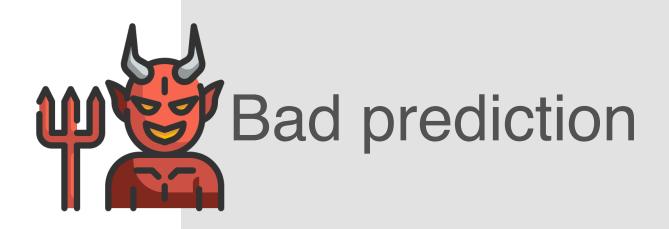
// p: prediction on the number of skiing days

Eirst dayskiing daysd: Actual number of skiing daysOPT buy)Truth: d < B (OPT rent)Advice: p < BALG(B, d, p) $= \frac{B}{B}$ OPT(B, d)

SKI-Rental with prediction (p)If $p \geq B$ Buy the ski on the first day else (p < B) Keep renting for all

Truth: $d \ge B$





- // p: prediction on the number of skiing days

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nting for all	skiing	days
		<i>d</i> : Actual number of skiing days
Truth: $d \ge B$ (O	PT buy)	Truth: $d < B$ (OPT rent)
Advice: $p \ge B$ ALG(B, d, p)	B	$\begin{array}{l} \text{Advice: } p < B \\ \text{ALG}(B, d, p) \end{array} d \end{array}$
OPT(B,d)	B	OPT(B,d) = d
Advice: <i>p</i> < <i>B</i> ALG(<i>B</i> , <i>d</i> , <i>p</i>)		

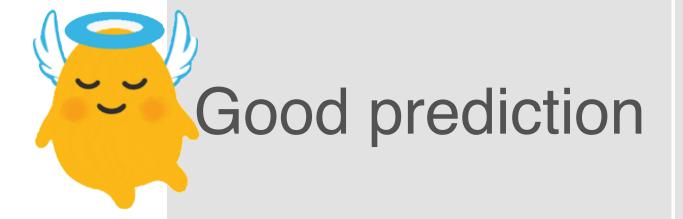
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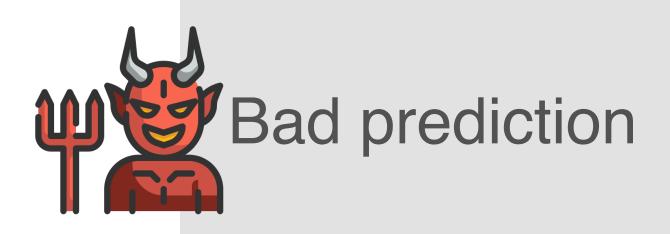
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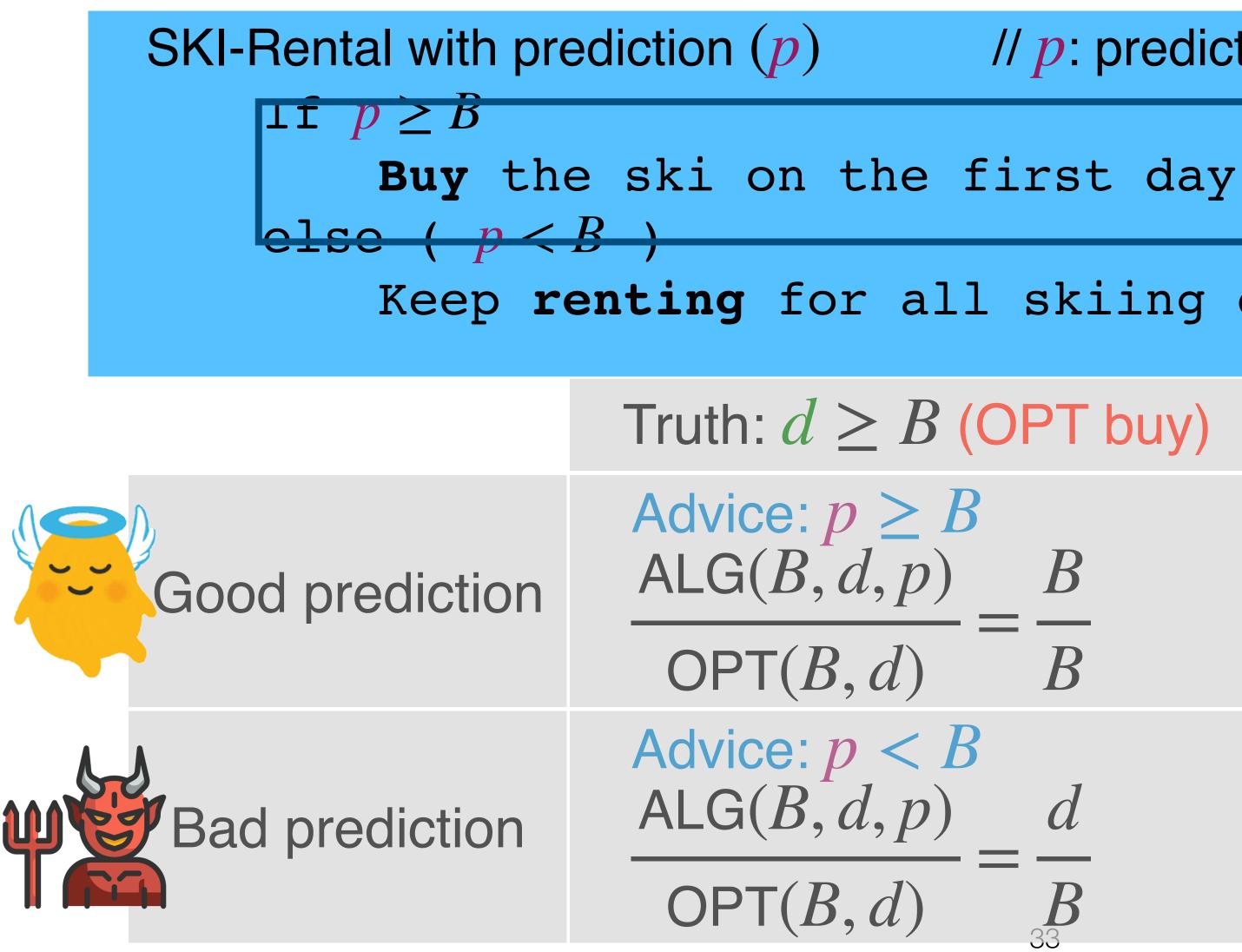
ALG(B, d, p)

OPT(B, d)





d: Actual number of skiing days Truth: d < B (OPT rent) Advice: $p \ge B$ Advice: p < B $\frac{ALG(B, d, p)}{OPT(B, d)} = \frac{d}{d}$ $\mathsf{ALG}(B,d,p) = B$ OPT(B, d) B ∞

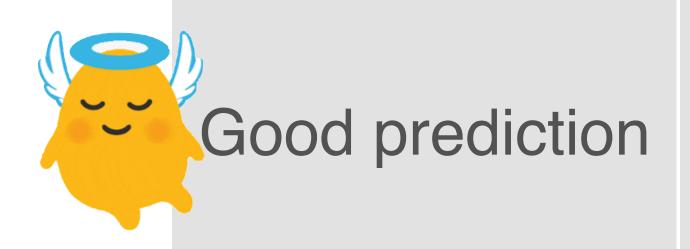


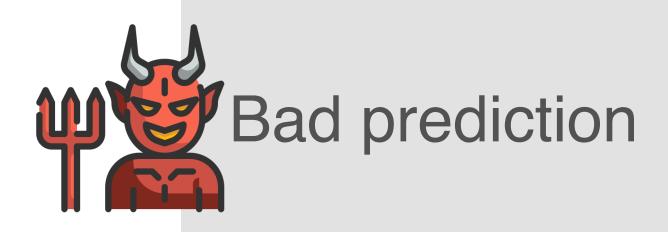
// p: prediction on the number of skiing days

l skiing d	days d: Actual number of skiing days
(OPT buy)	Truth: $d < B$ (OPT rent)
$B = \frac{B}{B}$	$\frac{\text{Advice: } p < B}{\text{ALG}(B, d, p)} = \frac{d}{d}$ $OPT(B, d) = d$
$B = \frac{d}{B}_{33}$	$\frac{\text{Advice: } p \ge B}{\text{ALG}(B, d, p)} = \frac{B}{d} \le \frac{B}{1} = B$ $OPT(B, d)$

SKI-Rental with prediction (p) If $p \geq B$ Buy the ski on the first day else (p < B) Keep renting for all skiing days

Truth: $d \geq B$ (OPT buy)





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$=\frac{d}{B}=\frac{\infty}{B}$	$\frac{\text{Advice: } p \ge B}{\text{ALG}(B, d, p)} = \frac{B}{d} \le \frac{B}{1} = B$ $OPT(B, d)$

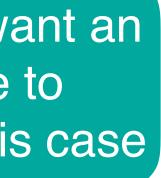
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(OPT buy)	Truth: $d < B$ (OPT rent)	
$B = \frac{B}{-} = 1$	Advice: $p < B$ ALG (B, d, p) d	
	OPT (<i>B</i> , <i>d</i>) To be robust, we want algorithm close	
$= \frac{d}{B} = \frac{\infty}{B}$	$\frac{\text{Advice: } p \ge B}{\text{ALG}(B, d, p)} = \frac{2}{d} \le \frac{2}{1} = B$ $OPT(B, d)$	



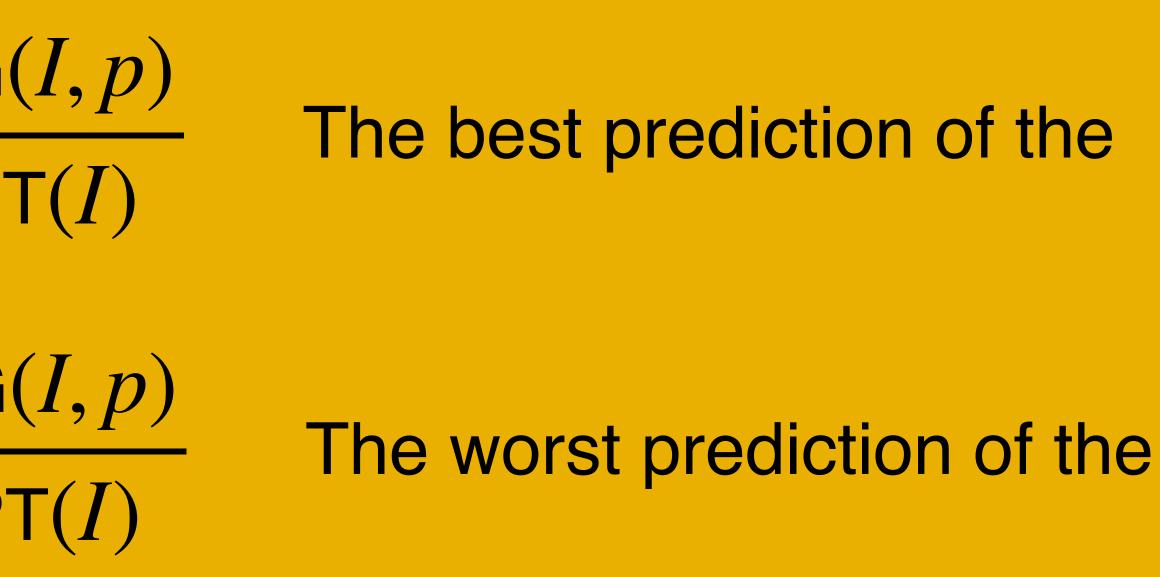
- Blindly trusting the prediction can cause disasters!

What Happened

• Especially when the prediction is wrong (that is, fail to be robust)

Consistency: max min I p OPT(I)input Robustness: max max P I = p P OPT(I)input

What Happened



SKI-Rental with prediction (p) If $p \geq B$ Buy the ski on the first day else (p < B) Keep **renting** until the *B*-th day

Ski Rental Algorithm 1.5

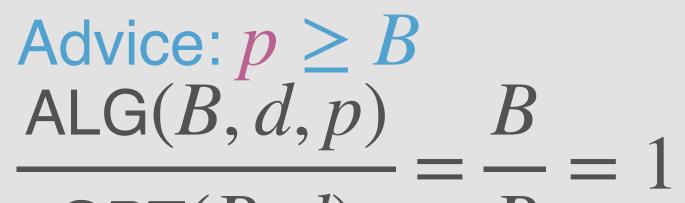
- // p: prediction on the number of skiing days

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Truth: $d \geq B$ (OPT buy)





OPT(B, d) B



- // p: prediction on the number of skiing days
- d: Actual number of skiing days Truth: d < B (OPT rent) Advice: p < B $\frac{ALG(B, d, p)}{OPT(B, d)} = \frac{d}{d} = 1$ Advice: $p \ge B$ $\frac{ALG(B, d, p)}{OPT(B, d)} = \frac{B}{d} \le \frac{B}{1} = B$

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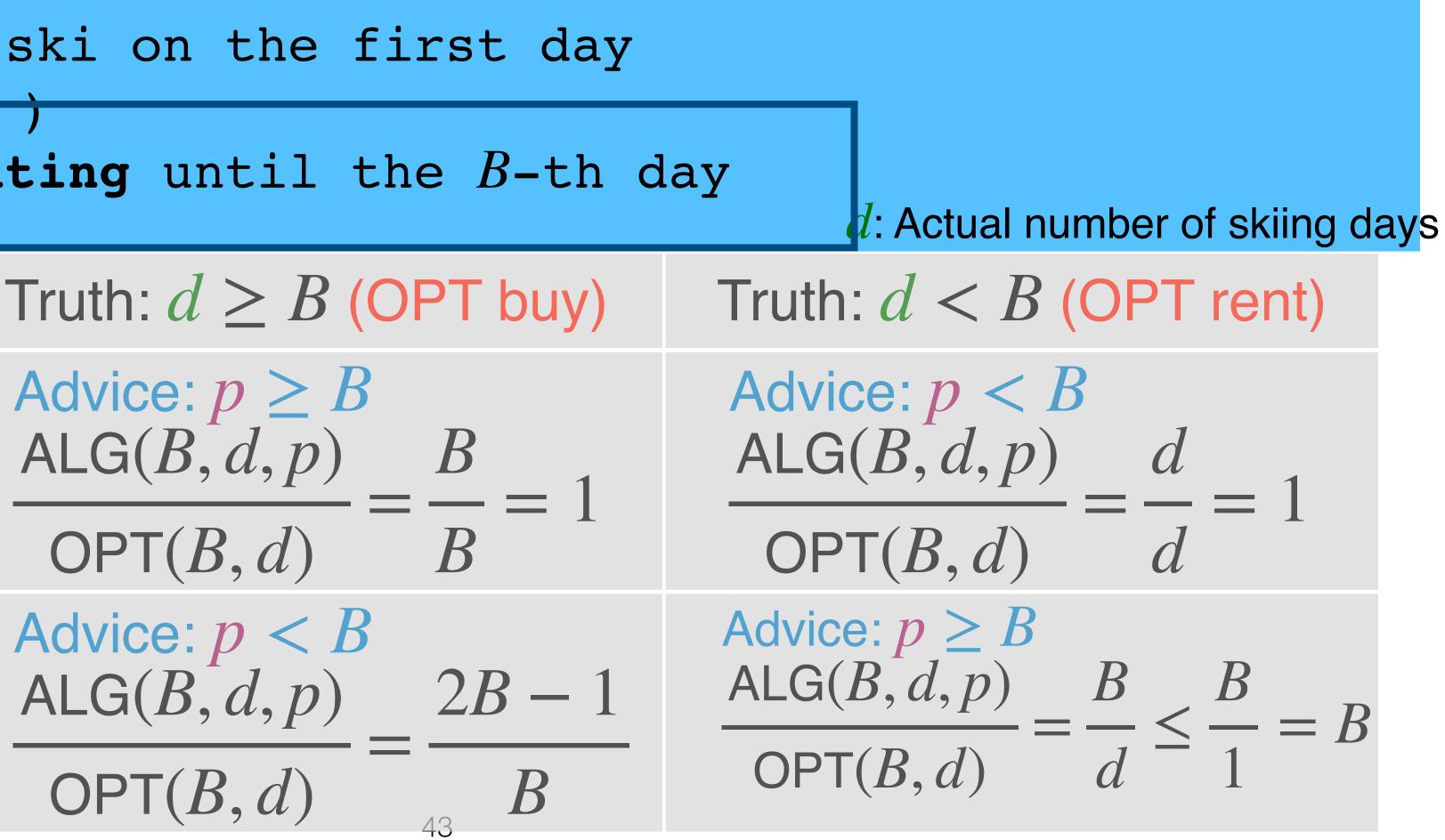
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ALG(B, d, p)

OPT(B, d)

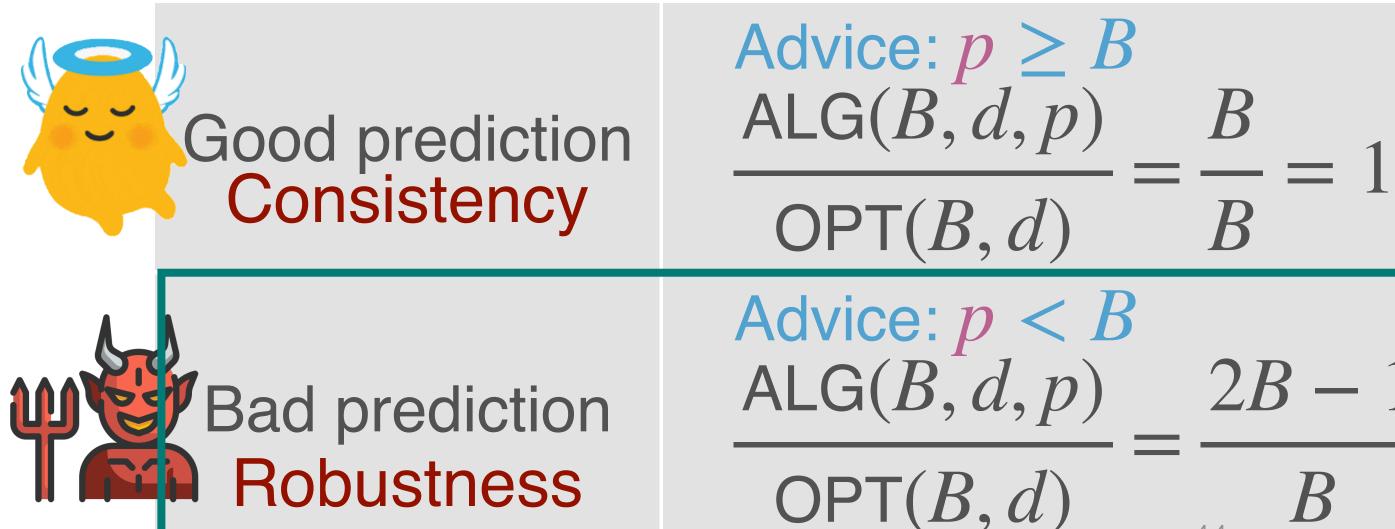




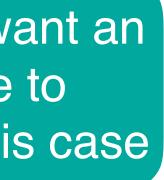


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- // p: prediction on the number of skiing days
- d: Actual number of skiing days Truth: d < B (OPT rent) Advice: p < BALG(B, d, p) dTo be robust, we want an OPT(B, d)algorithm close to 2-competitive in this case Advice: $p \ge B$ ALG(B, d, p)2B - 1- < - = BOPT(B, d)B



- The robustness is still bad when the prediction is wrong

What Happened

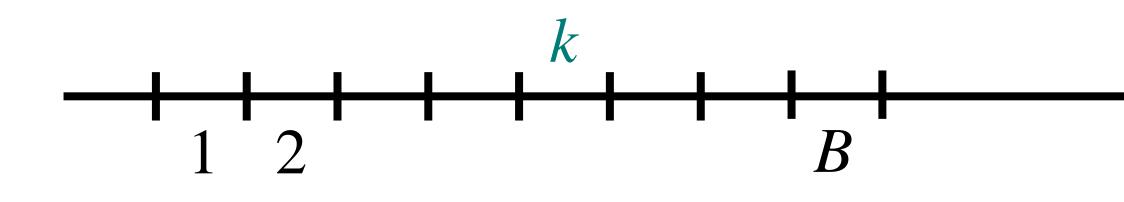
Especially when the algorithm is tricked into buying the ski

Trustness Parameter

- advice
 - $k \in [1,B]$
 - k = 1: the algorithm fully trusts the advice
 - k = B: the algorithm does not trust the advice at all

• We introduce a parameter k to indicate how much the algorithm trust the

SKI-Rental with prediction (p, k)If $p \geq B$ Keep renting until the k-th day else (p < B) Keep renting until the *B*-th day



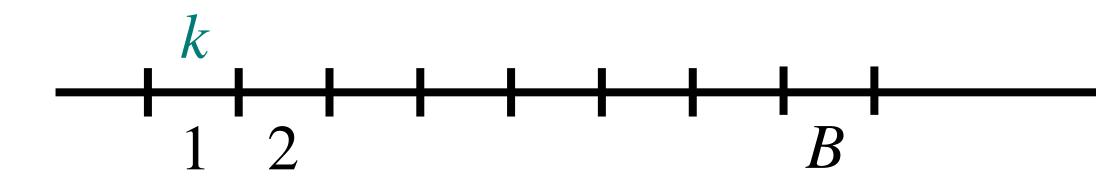
// k is our "trust parameter"

x-th Day



SKI-Rental with prediction (p, k)If $p \geq B$ Keep renting until the k-th day else (p < B) Keep renting until the B-th day

Fully trust the prediction



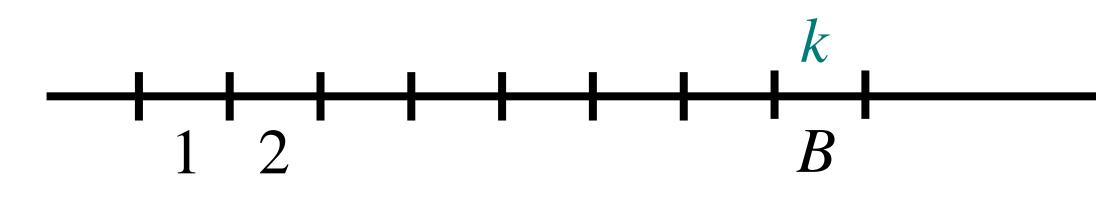
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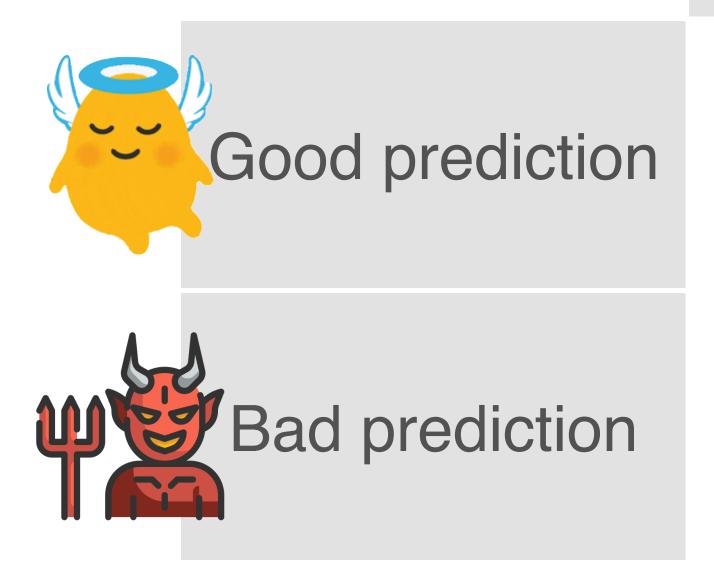


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ang until the k-th day// k is our "trust parameter"ang until the B-th dayd: Actual number of skiing daysTruth: $d \ge B$ (OPT buy)Truth: d < B (OPT rent)

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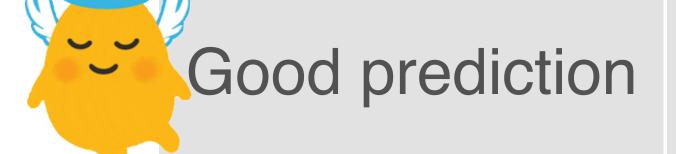


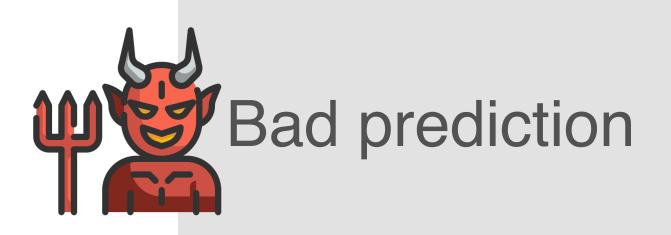


day// k is our "trust parameter"dayd: Actual number of skiing daysOPT buy)Truth: d < B (OPT rent)

SKI-Rental with prediction (p, k)If $p \ge B$ Keep renting until the k-th c else (p < B) Keep renting until the B-th c

Truth: $d \ge B$ (Advice: $p \ge B$





day	// k is our "trust parameter"
day	d: Actual number of skiing days
(OPT buy)	Truth: $d < B$ (OPT rent)
3	Advice: $p < B$

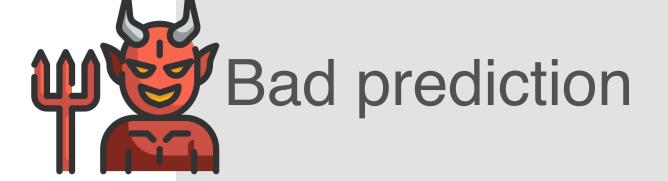
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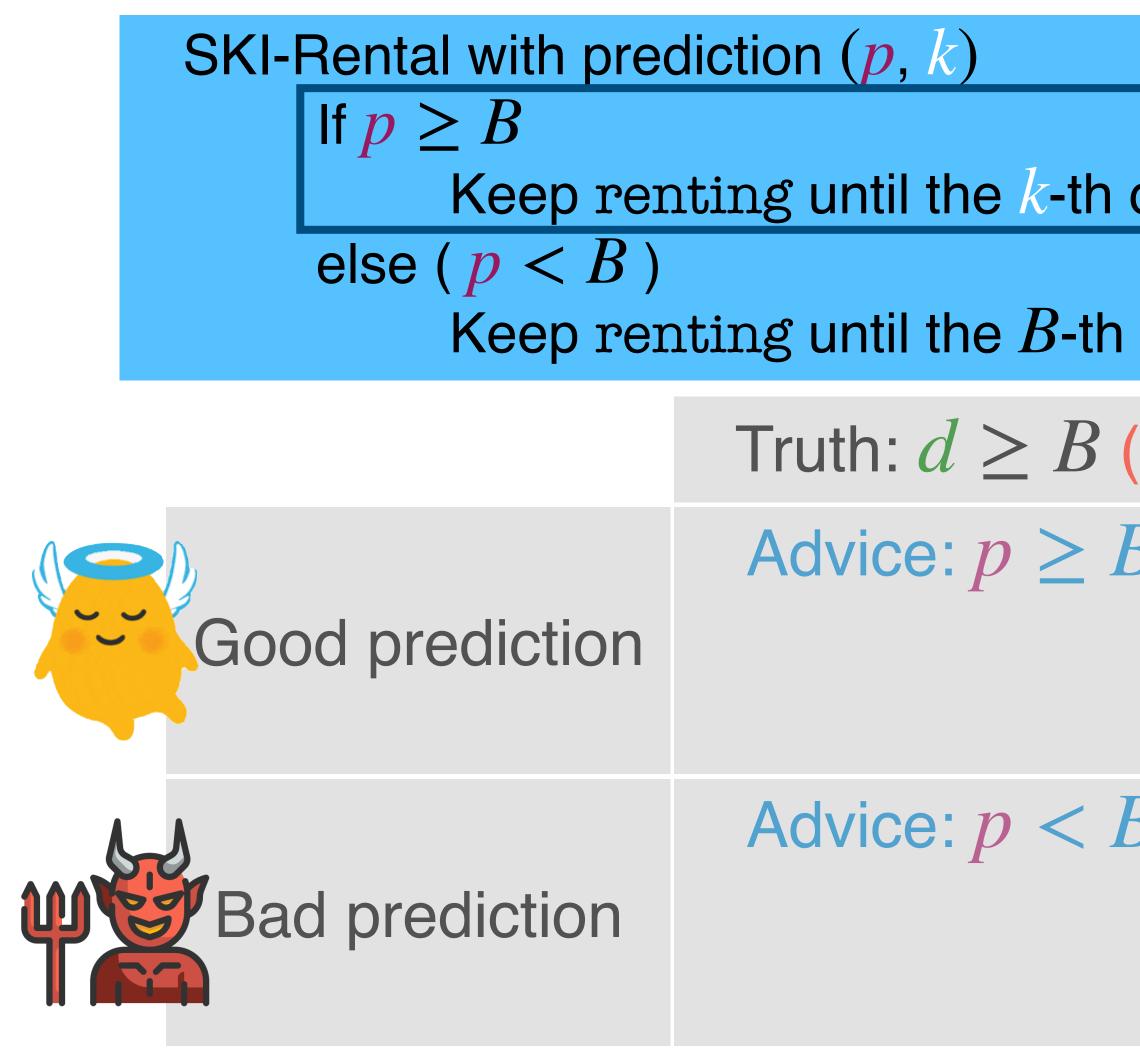
Advice: $p \ge E$

Good prediction

Advice: p < k



day	// k is our "trust parameter"
day	d: Actual number of skiing days
(OPT buy)	Truth: $d < B$ (OPT rent)
8	Advice: <i>p</i> < <i>B</i>
8	Advice: $p \geq B$

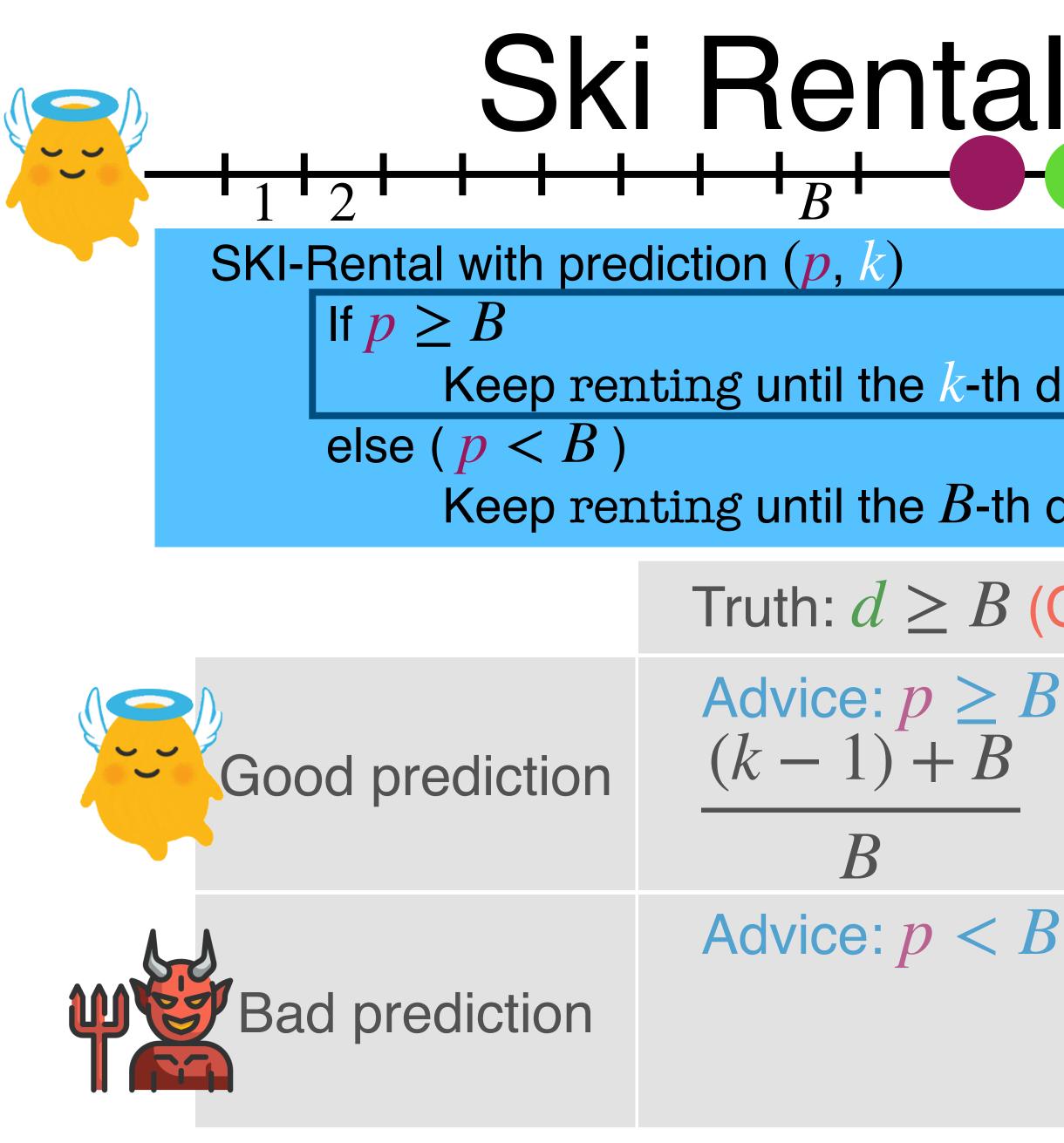


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OPT buy)	Truth: $d < B$ (OPT rent)
3	Advice: <i>p</i> < <i>B</i>
3	Advice: $p \ge B$





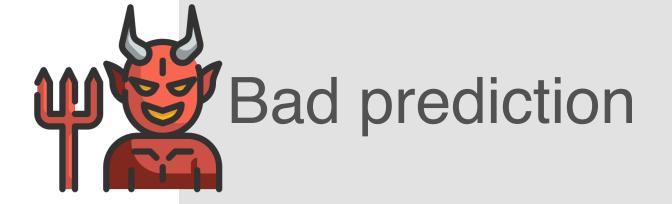
day	// k is our "trust parameter"
day	d: Actual number of skiing days
OPT buy)	Truth: $d < B$ (OPT rent)
3	Advice: <i>p</i> < <i>B</i>
3	Advice: $p \ge B$



SKI-Rental with prediction (p, k)If $p \ge B$ Keep renting until the k-th c else (p < B) Keep renting until the B-th c

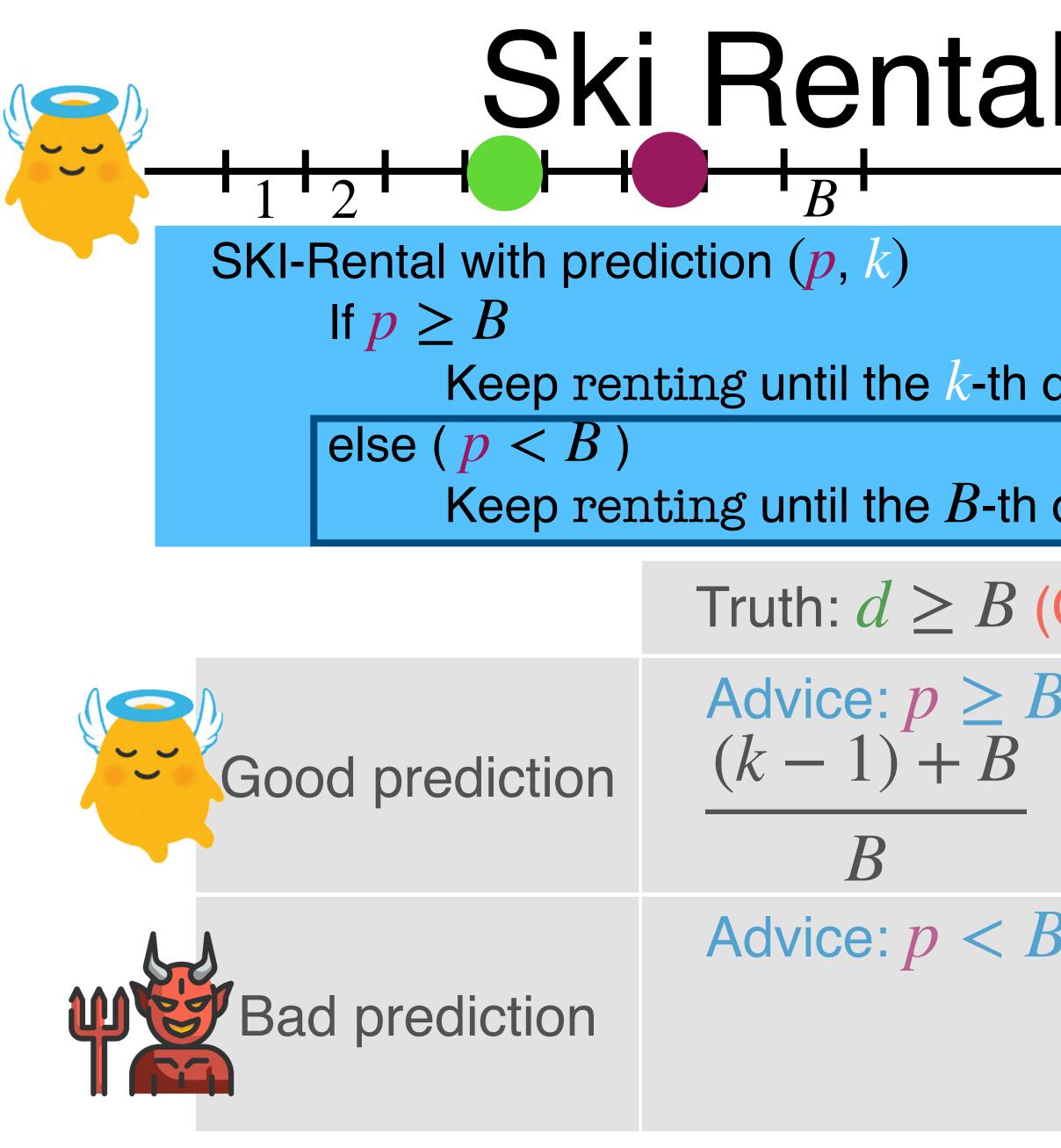
- Truth: $d \ge B$ (
- Advice: $p \ge E$ (k-1) + B

Advice: p < 1



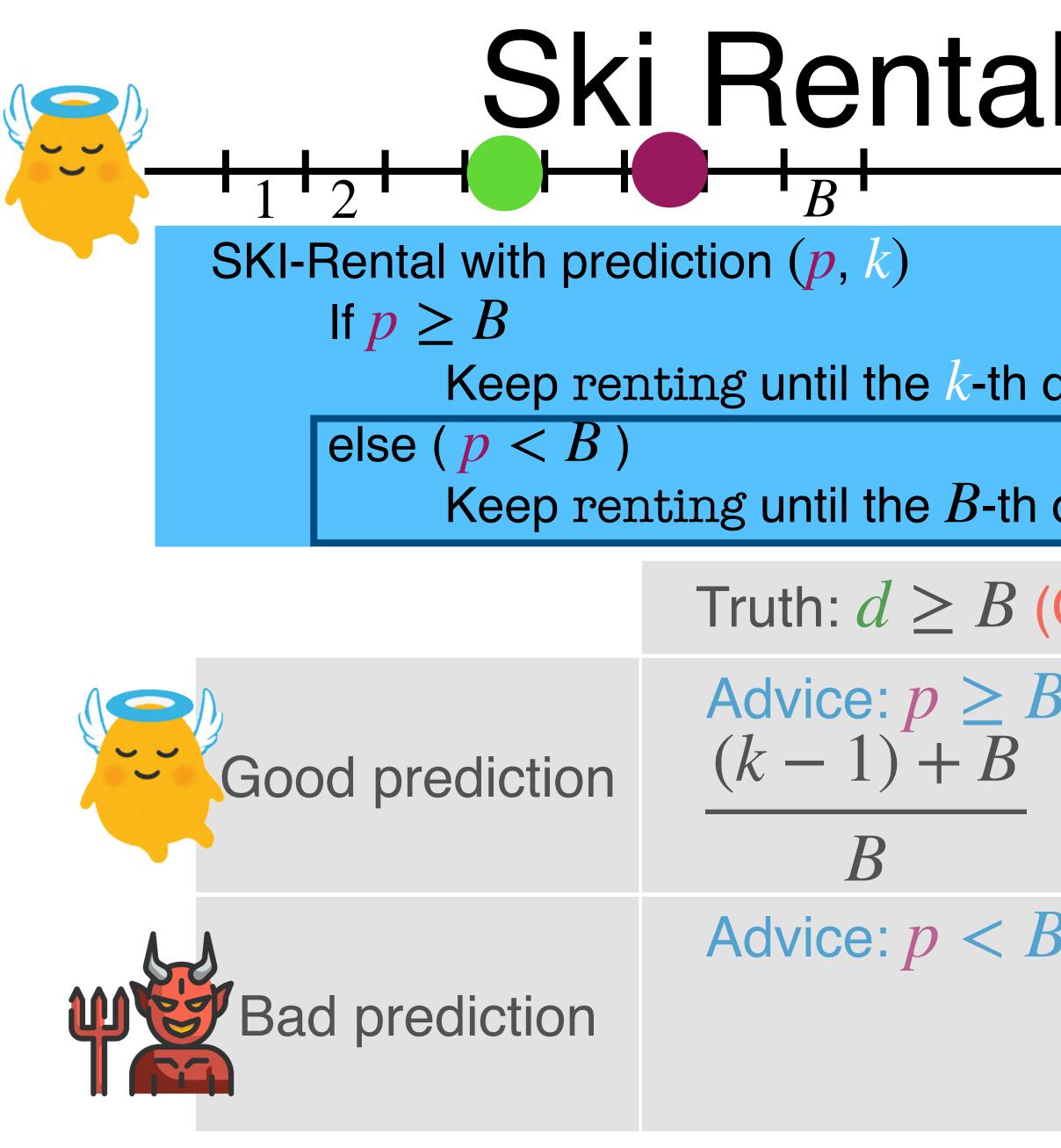
Good prediction

day day	<pre>// k is our "trust parameter" d: Actual number of skiing days</pre>
(OPT buy) B	Truth: $d < B$ (OPT rent) Advice: $p < B$
B	Advice: $p \ge B$



day day	<pre>// k is our "trust parameter" d: Actual number of skiing days</pre>
(OPT buy)	Truth: $d < B$ (OPT rent)
3	Advice: <i>p</i> < <i>B</i>
3	Advice: $p \geq B$





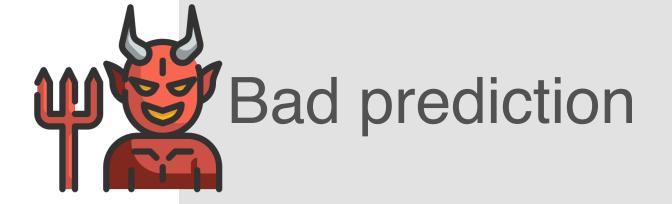
day day	 <i>I k</i> is our "trust parameter" <i>d</i>: Actual number of skiing days
(OPT buy)	Truth: $d < B$ (OPT rent)
3	$\begin{array}{l} \text{Advice: } p < B \\ d \\ \overline{d} \end{array}$
3	Advice: $p \ge B$



SKI-Rental with prediction (p, k)If $p \ge B$ Keep renting until the k-th c else (p < B) Keep renting until the B-th c

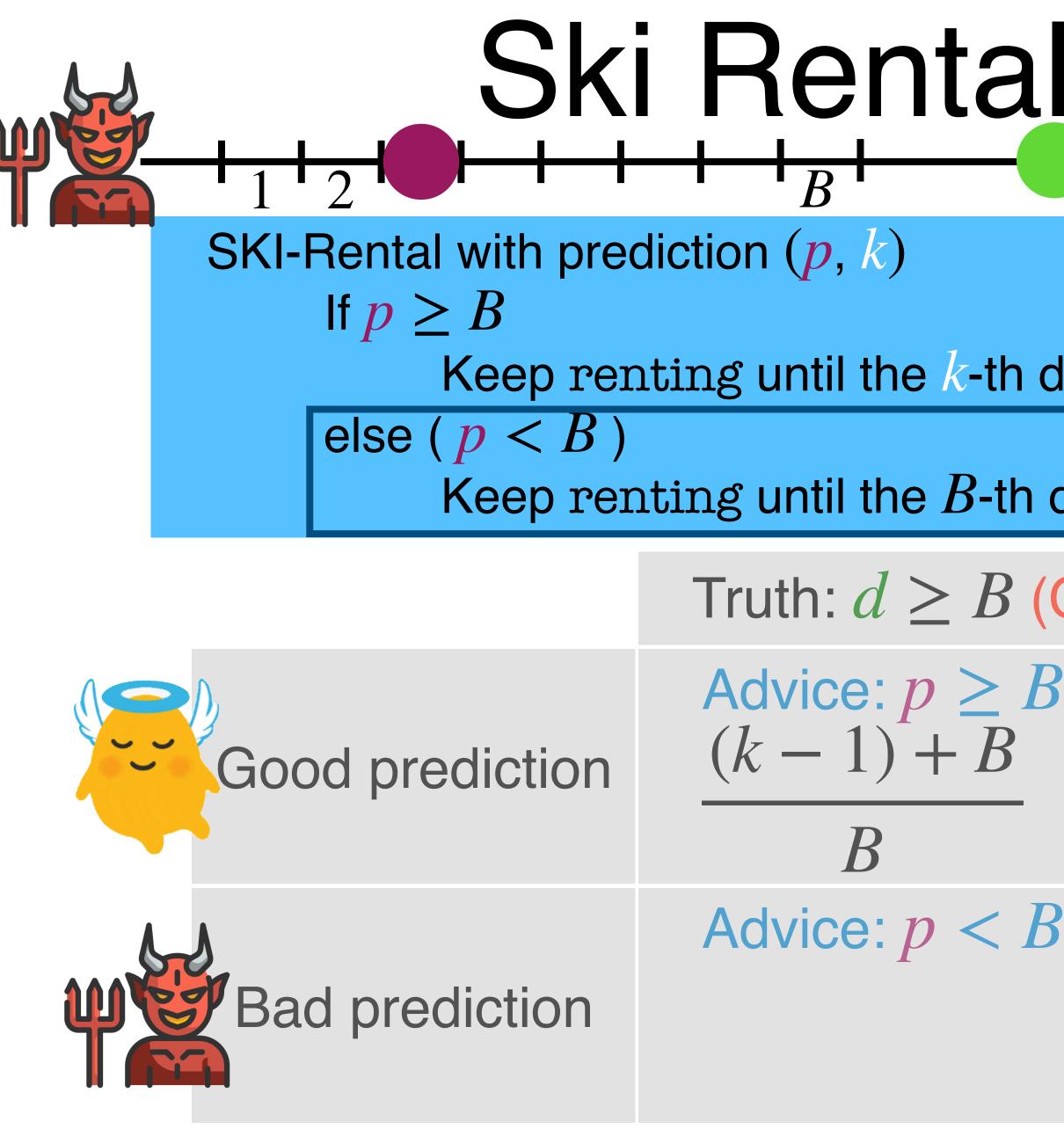
- Truth: $d \ge B$ (
- Advice: $p \ge E$ (k-1) + B

Advice: p < 1



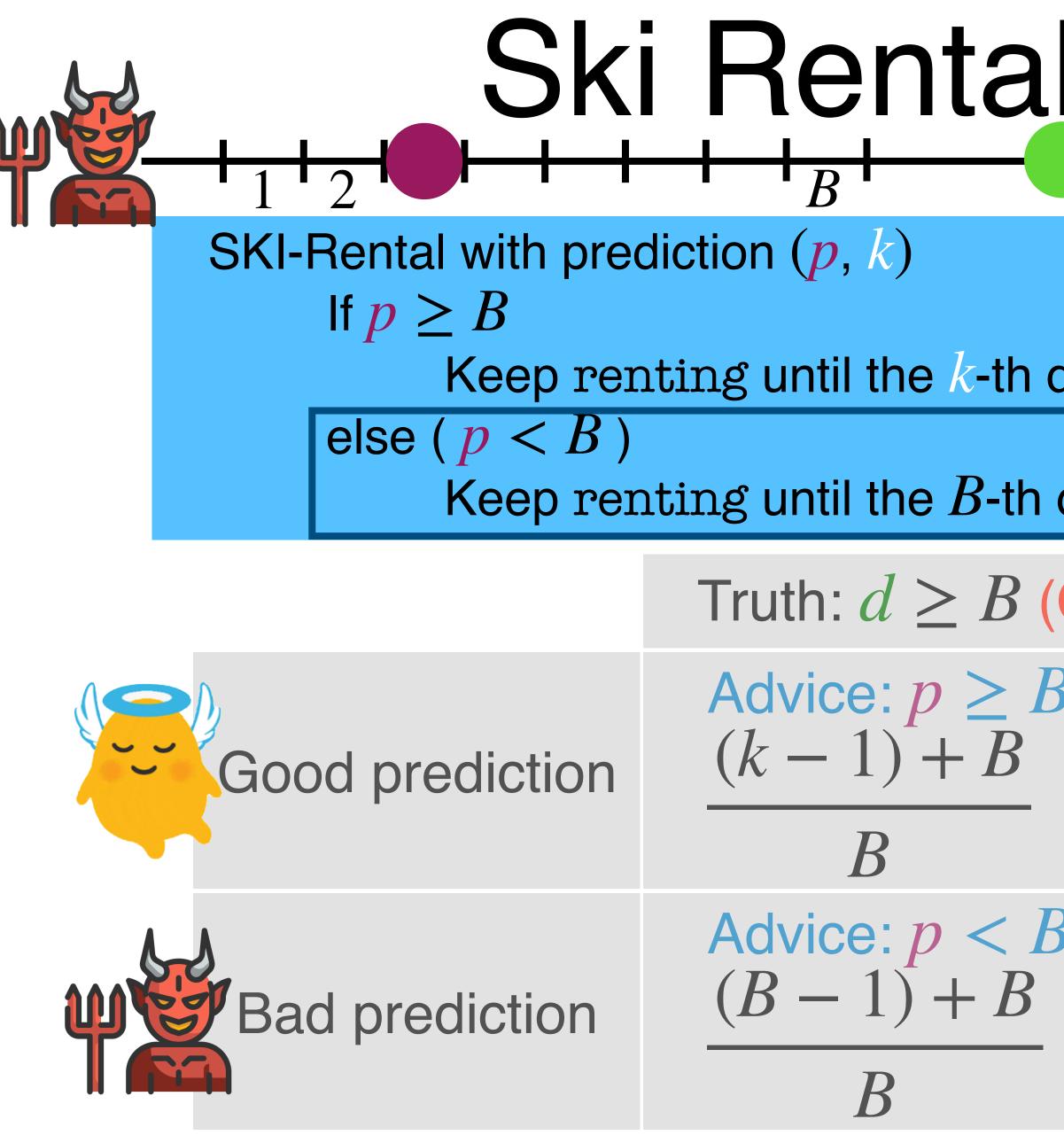
Good prediction

day day	 // k is our "trust parameter" d: Actual number of skiing days
(OPT buy)	Truth: $d < B$ (OPT rent) Advice: $p < B$ $\frac{d}{d}$
B	Advice: $p \ge B$



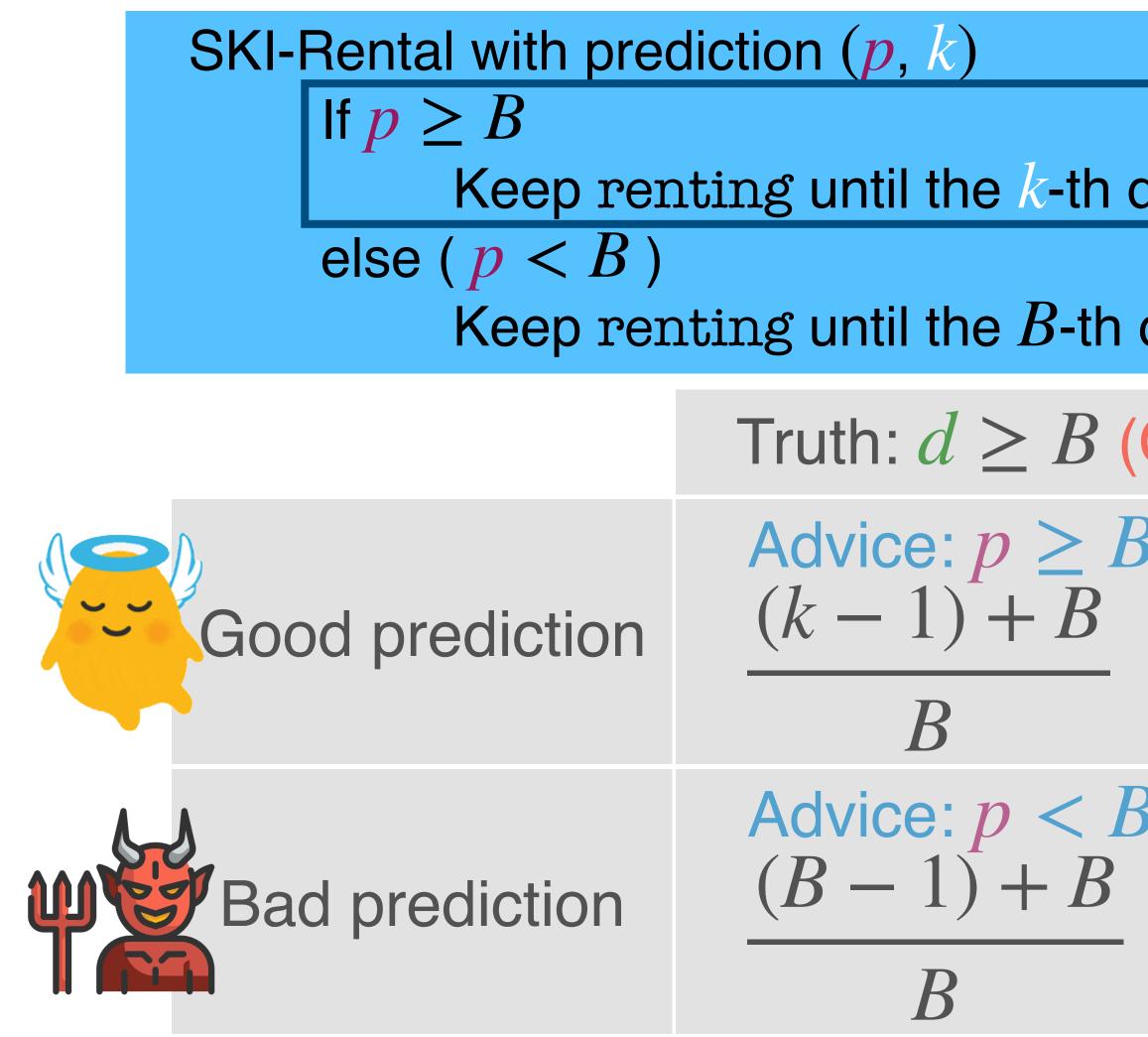
day day	 // k is our "trust parameter" d: Actual number of skiing days
(OPT buy)	Truth: $d < B$ (OPT rent)
3	$\begin{array}{l} \text{Advice: } p < B \\ d \\ \overline{d} \end{array}$
3	Advice: $p \ge B$



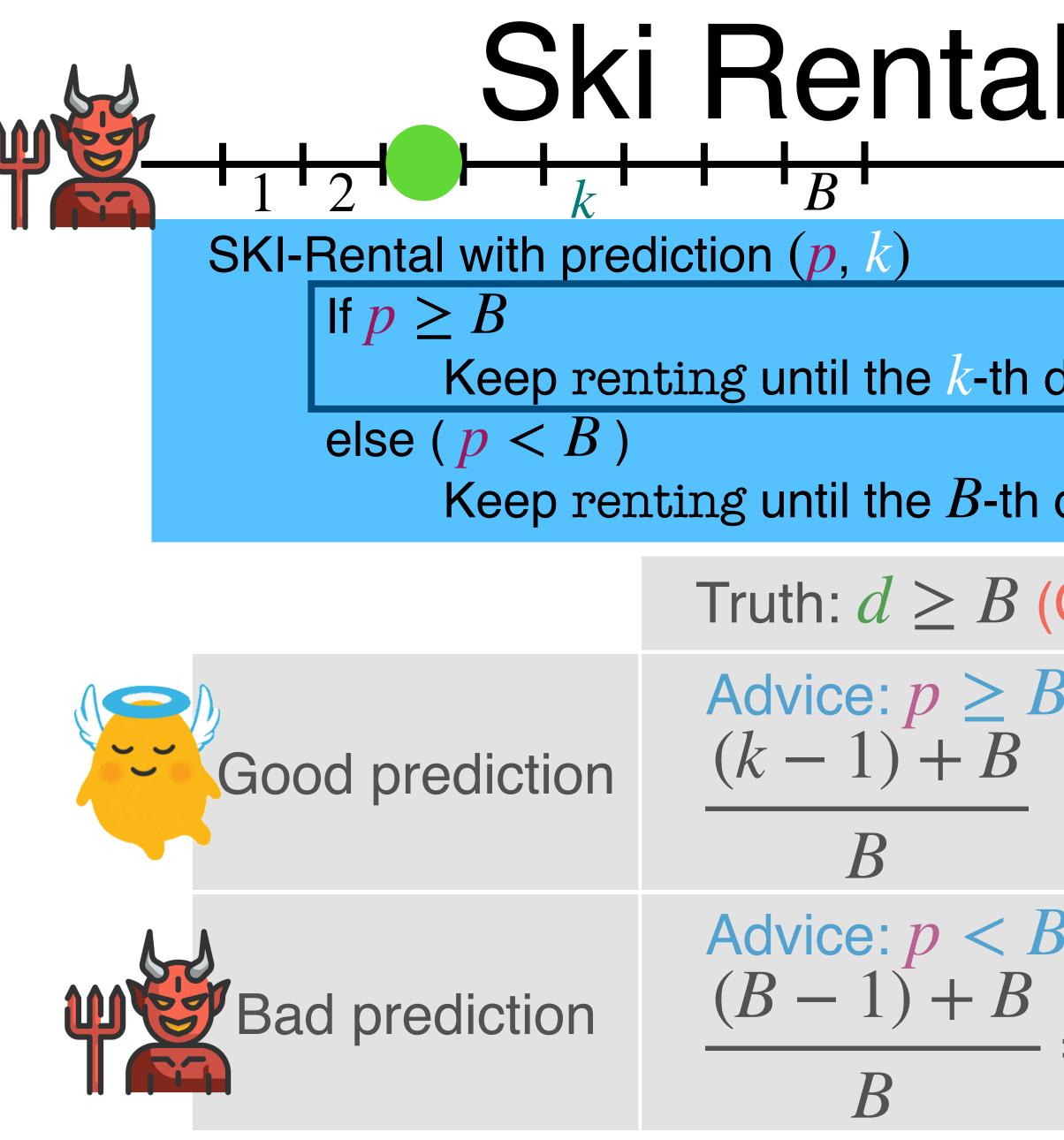


day day	<pre>// k is our "trust parameter" d: Actual number of skiing days</pre>
(OPT buy)	Truth: $d < B$ (OPT rent)
8	$\begin{array}{l} \text{Advice: } p < B \\ d \\ \overline{d} \end{array}$
$B = 2 - \frac{1}{B}$	Advice: $p \geq B$



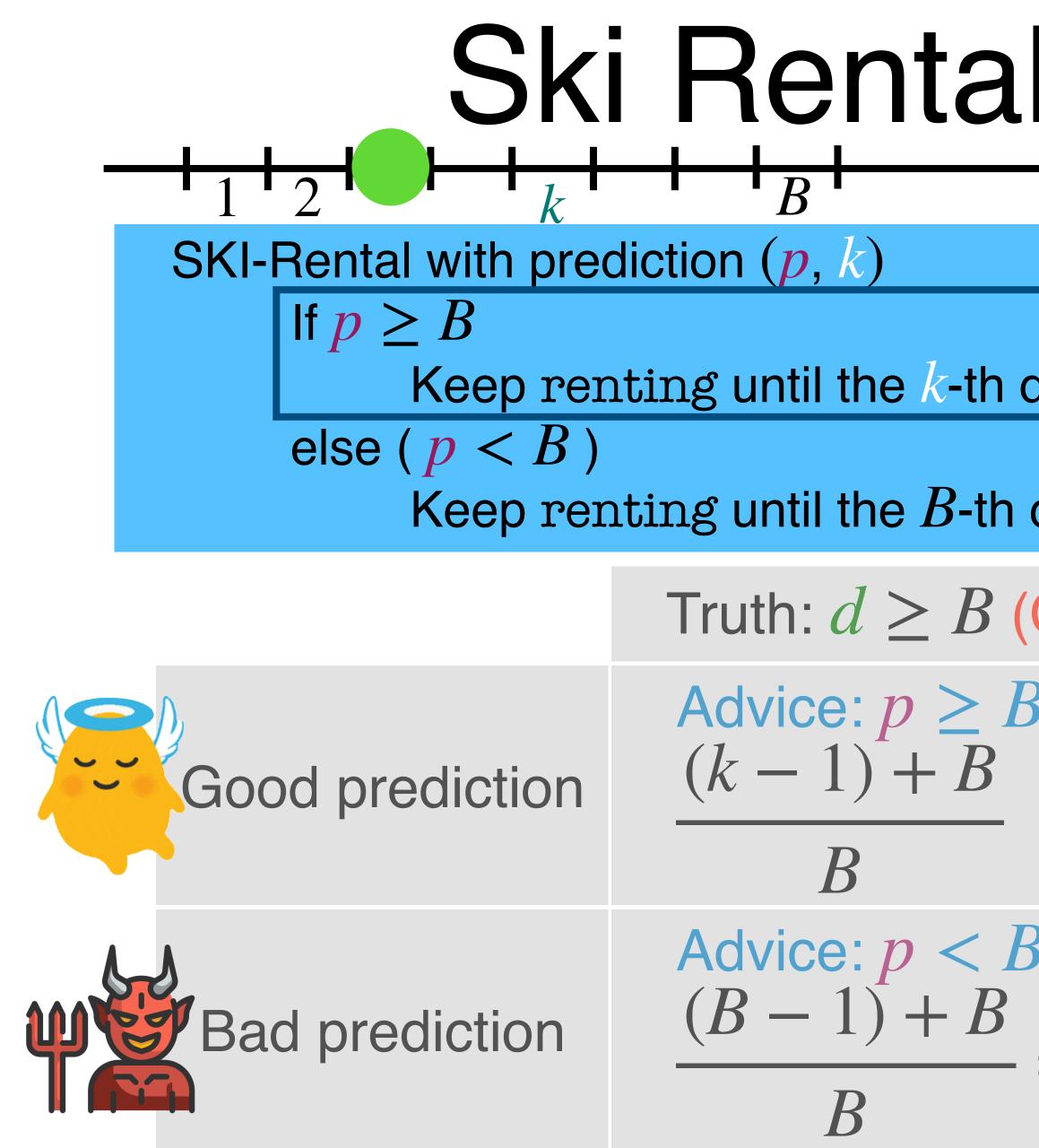


day	// k is our "trust parameter"
day	d: Actual number of skiing days
(OPT buy) B	Truth: $d < B$ (OPT rent) Advice: $p < B$ $\frac{d}{d}$
$B = 2 - \frac{1}{B}$	Advice: $p \ge B$



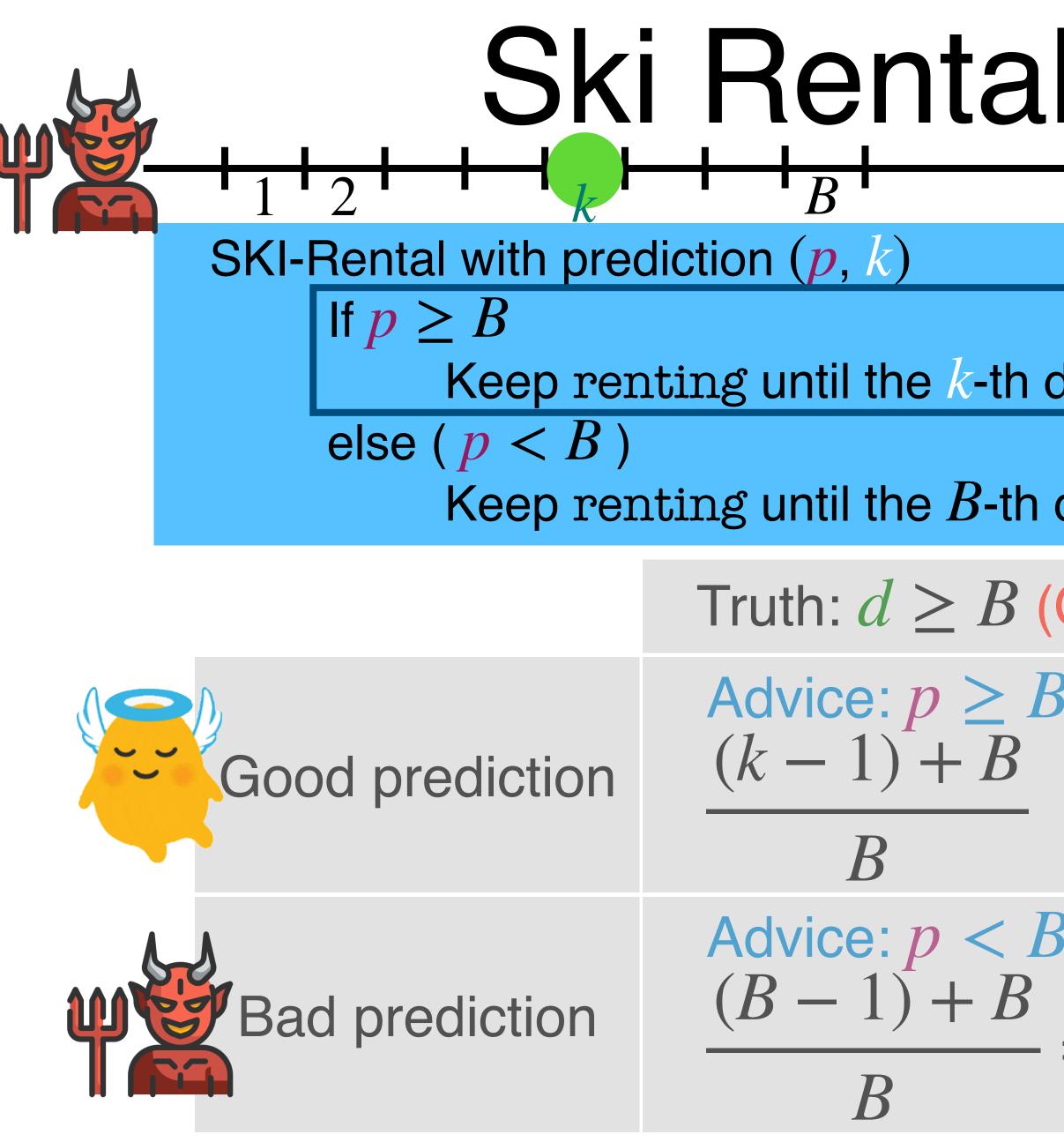
day day	 <i>I k</i> is our "trust parameter" <i>d</i>: Actual number of skiing days 	
(OPT buy)	Truth: $d < B$ (OPT rent)	
3	$\begin{array}{l} \text{Advice: } p < B \\ \frac{d}{d} \end{array}$	
$= 2 - \frac{1}{B}$	Advice: $p \ge B$	





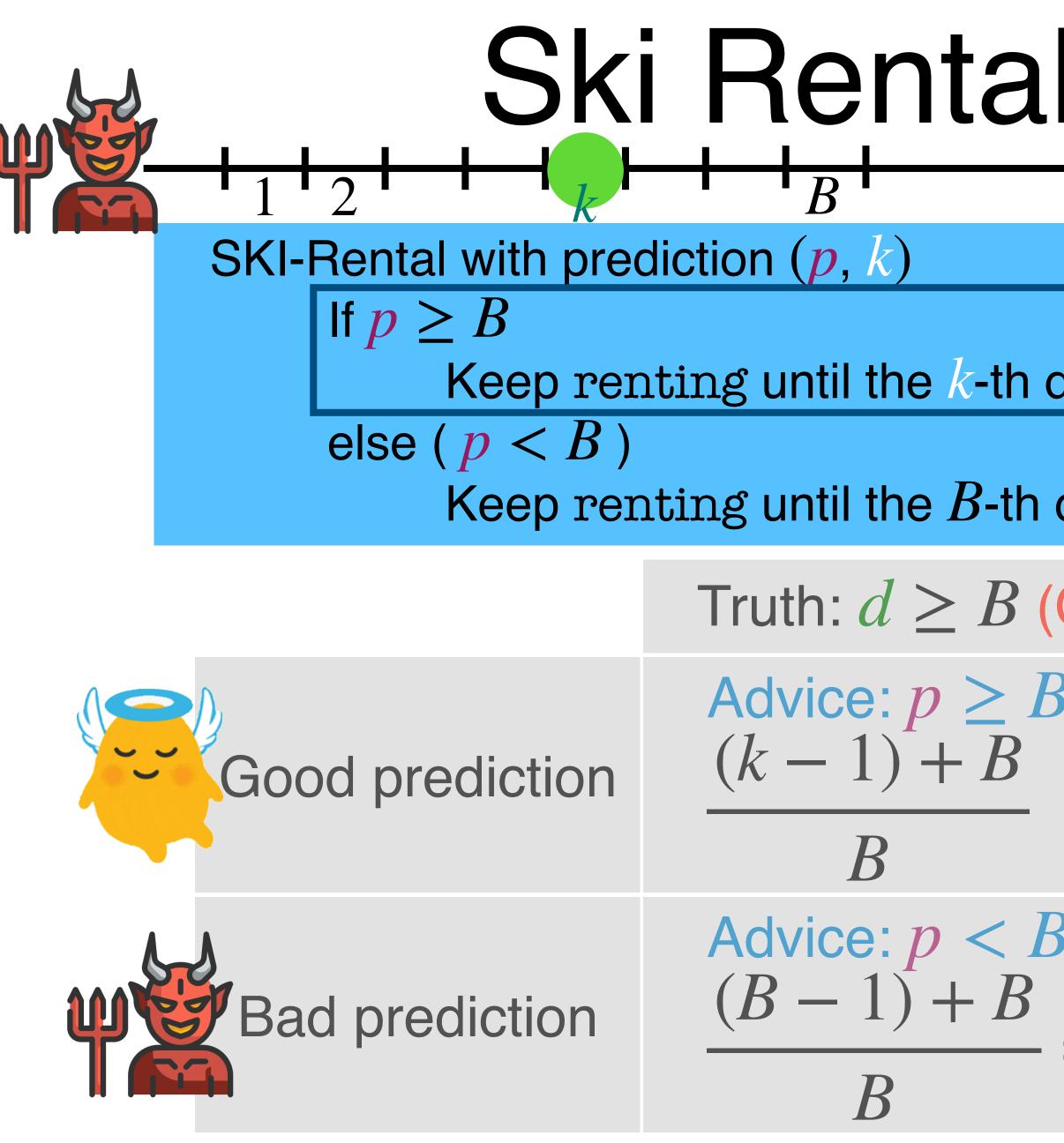
day day	 <i>I k</i> is our "trust parameter" <i>d</i>: Actual number of skiing days
(OPT buy)	Truth: $d < B$ (OPT rent)
3	$\begin{array}{l} \text{Advice: } p < B \\ d \\ \overline{d} \end{array}$
$= 2 - \frac{1}{B}$	$\begin{array}{l} \text{Advice: } p \geq B \\ d \\ \overline{d} \end{array}$





day day	 <i>I k</i> is our "trust parameter" <i>d</i>: Actual number of skiing days
(OPT buy)	Truth: $d < B$ (OPT rent)
3	$\begin{array}{l} \text{Advice: } p < B \\ d \\ \overline{d} \end{array}$
$= 2 - \frac{1}{B}$	Advice: $p \ge B$





day day	<pre>// k is our "trust parameter" d: Actual number of skiing days</pre>
(OPT buy)	Truth: $d < B$ (OPT rent)
3	$\begin{array}{l} \text{Advice: } p < B \\ d \\ \overline{d} \end{array}$
$= 2 - \frac{1}{B}$	$\frac{\text{Advice: } p \ge B}{(k-1)+B} = 1 + \frac{B-1}{k}$



SKI-Rental with prediction (p, k)If $p \ge B$ Keep renting until the k-th c else (p < B) Keep renting until the B-th c

Truth:
$$d \ge B$$
 (

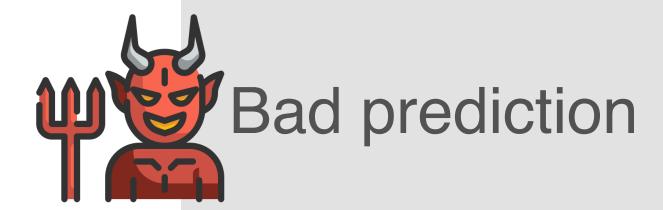
Advice:
$$p \ge B$$

 $(k-1) + B$

B

Advice: *p* < (B - 1) + B

B



Good prediction

day	// k is our "trust parameter"
day	d: Actual number of skiing days
(OPT buy)	Truth: $d < B$ (OPT rent)
$= 1 + \frac{k-1}{B}$	Advice: $p < B$ $\frac{d}{d} = 1$
$B = 2 - \frac{1}{B}$	$\frac{\text{Advice: } p \ge B}{(k-1)+B} = 1 + \frac{B-1}{k}$

SKI-Rental with prediction (p, k)If $p \ge B$ Keep renting until the *k*-th of the else (p < B)Keep renting until the *B*-th of the else (p < B)

Truth:
$$d \ge B$$
 (

Advice:
$$p \ge B$$

 $(k-1) + B$

B

Advice: p < B(B - 1) + B

В



Good prediction Consistency

day	// k is our "trust parameter"
day	d: Actual number of skiing days
(OPT buy)	Truth: $d < B$ (OPT rent)
$= 1 + \frac{k-1}{B}$	Advice: $p < B$ $\frac{d}{d} = 1$
$\frac{B}{B} = 2 - \frac{1}{B}$	$\frac{\text{Advice: } p \ge B}{(k-1)+B} = 1 + \frac{B-1}{k}$

What Happened

- Use a trust parameter to partially trust the prediction
 - In this case, trust parameter $k \in [1,B]$
 - The smaller k is, the more the algorithm trusts the prediction
- ALG2 is $(1 + \frac{k-1}{B})$ -consiste
 - When k = 1, the consistency is 1 and the robustness is B
 When k = B, the consistency and the robustness are both
 - When k = B, the consisten $2 \frac{1}{B}$

ant and
$$(1 + \frac{B-1}{k})$$
-robust

Prediction Error η

Prediction Error η

• Absolute error $\eta_1 = |p - d|$

Prediction Error η

- Absolute error $\eta_1 = |p d|$
- Squared error $\eta_2 = |p d|^2$

Prediction Error n

- Absolute error $\eta_1 = |p d|$
- Squared error $\eta_2 = |p d|^2$
- Classification error $\eta_c = 1$ if $p \neq d$
-

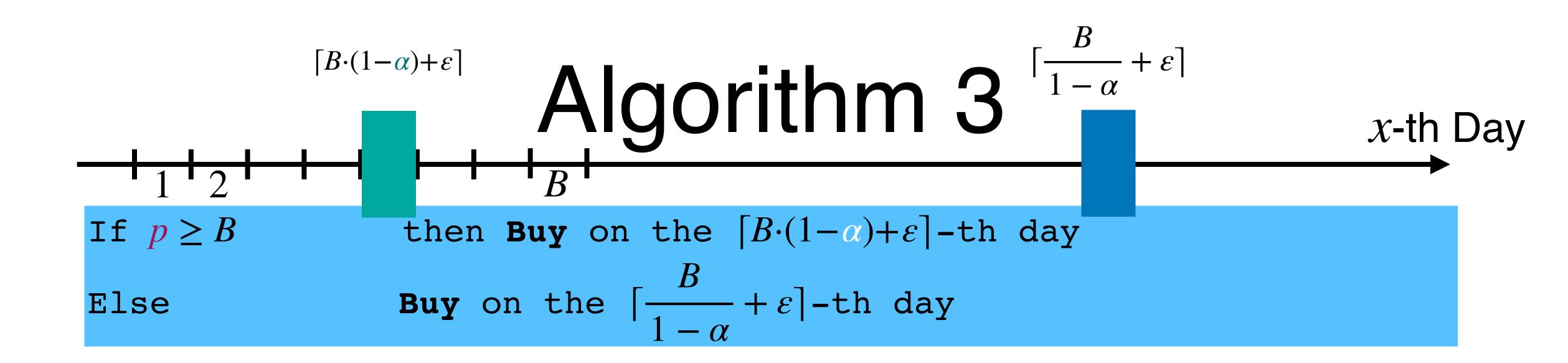
Ski Rental Algorithms Revisit: $\eta_1 = |p - d|$

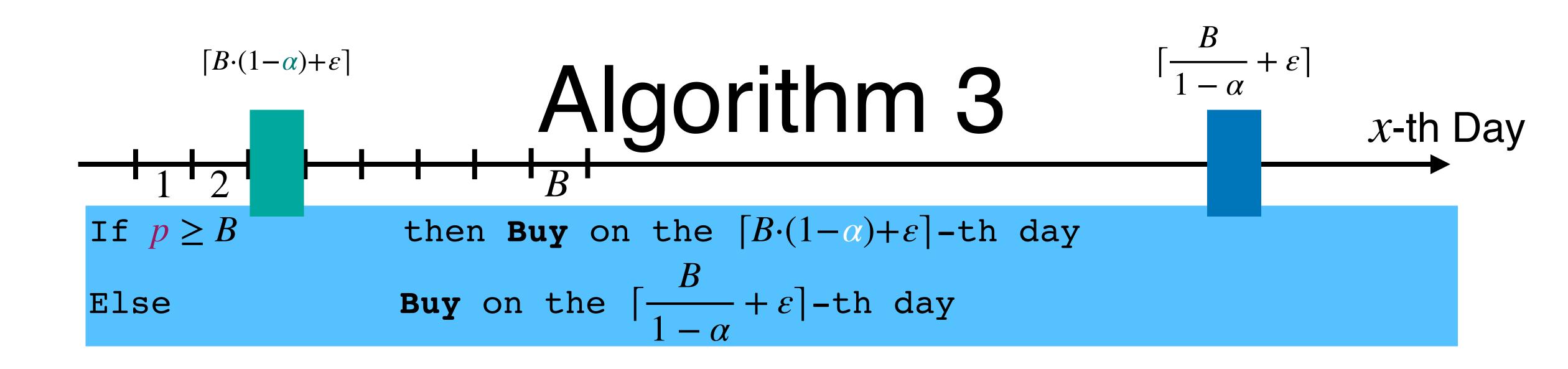
• $\eta_1 = |p - d|$

Algorithm 3

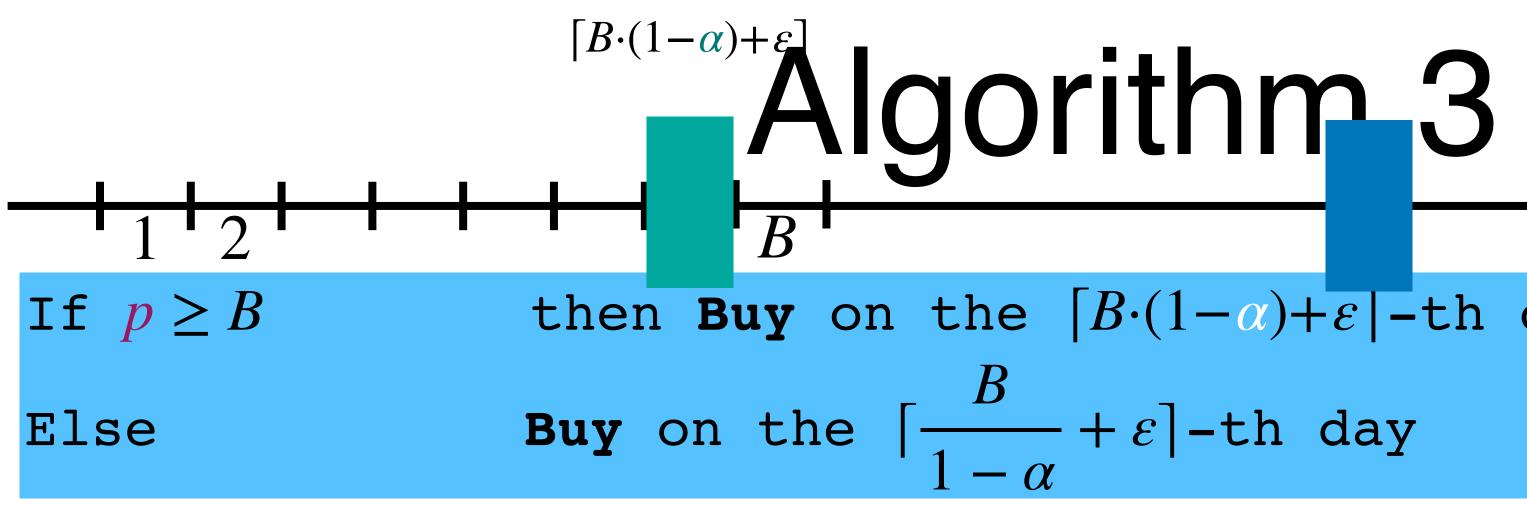
If $p \geq B$	then Buy on the
Else	Buy on the $\lceil \frac{B}{1-\alpha} \rceil$
LT2C	Buy on the $1-\alpha$

$[B \cdot (1-\alpha) + \varepsilon] - \text{th day}$ -+ \varepsilon] - \text{th} day





Big α

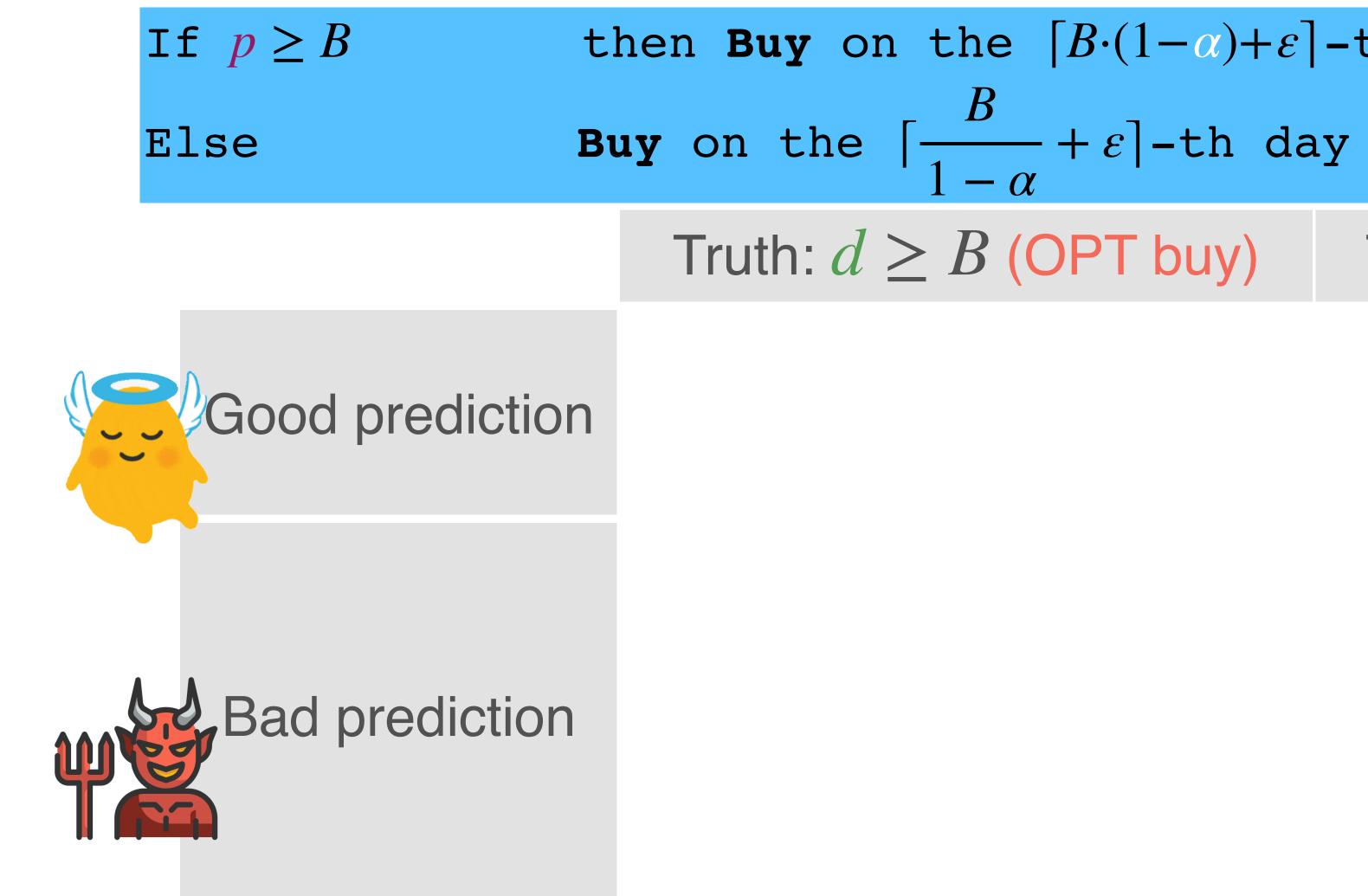


x-th Day then **Buy** on the $[B \cdot (1-\alpha) + \varepsilon]$ -th day

Small α

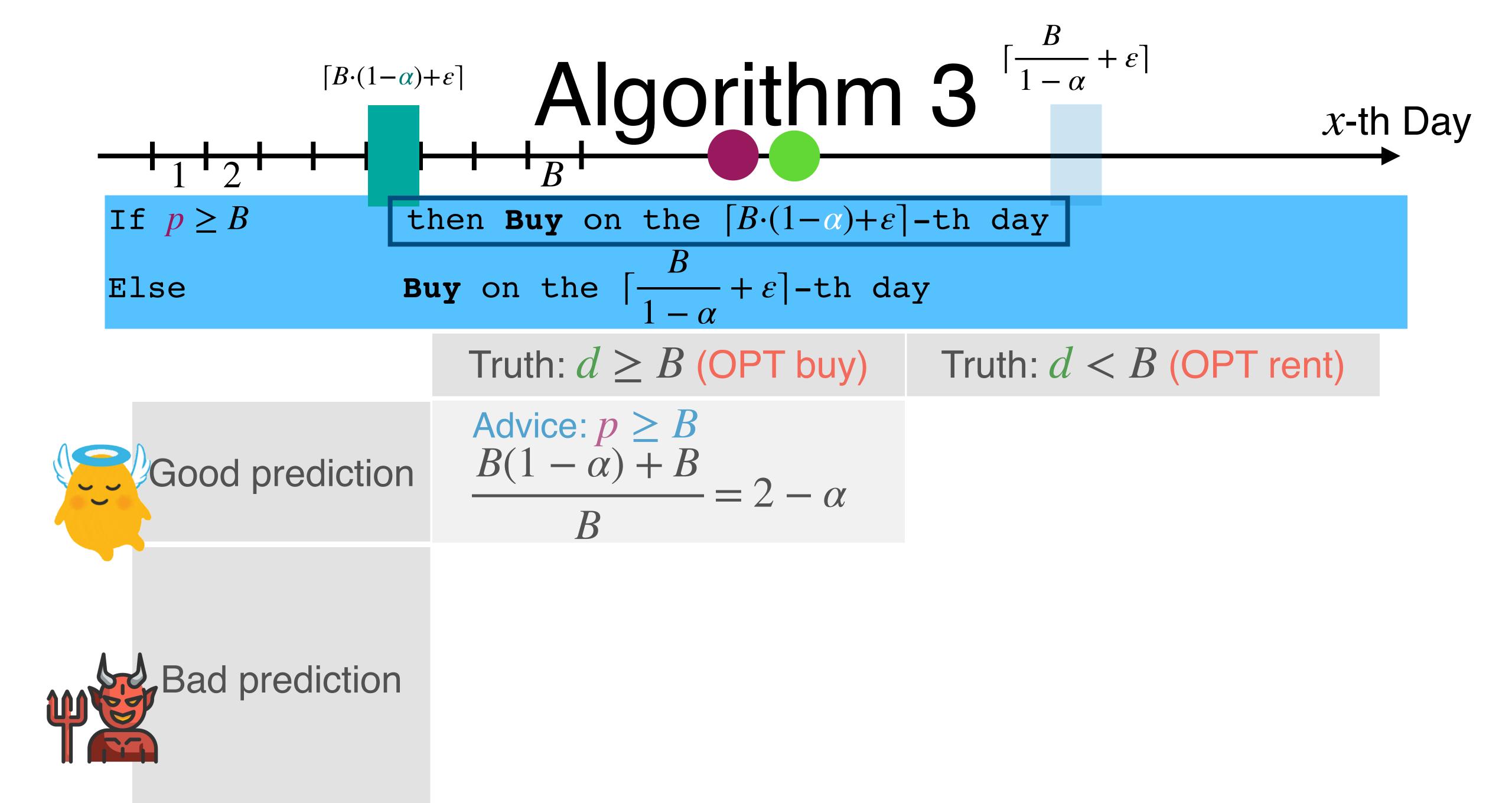


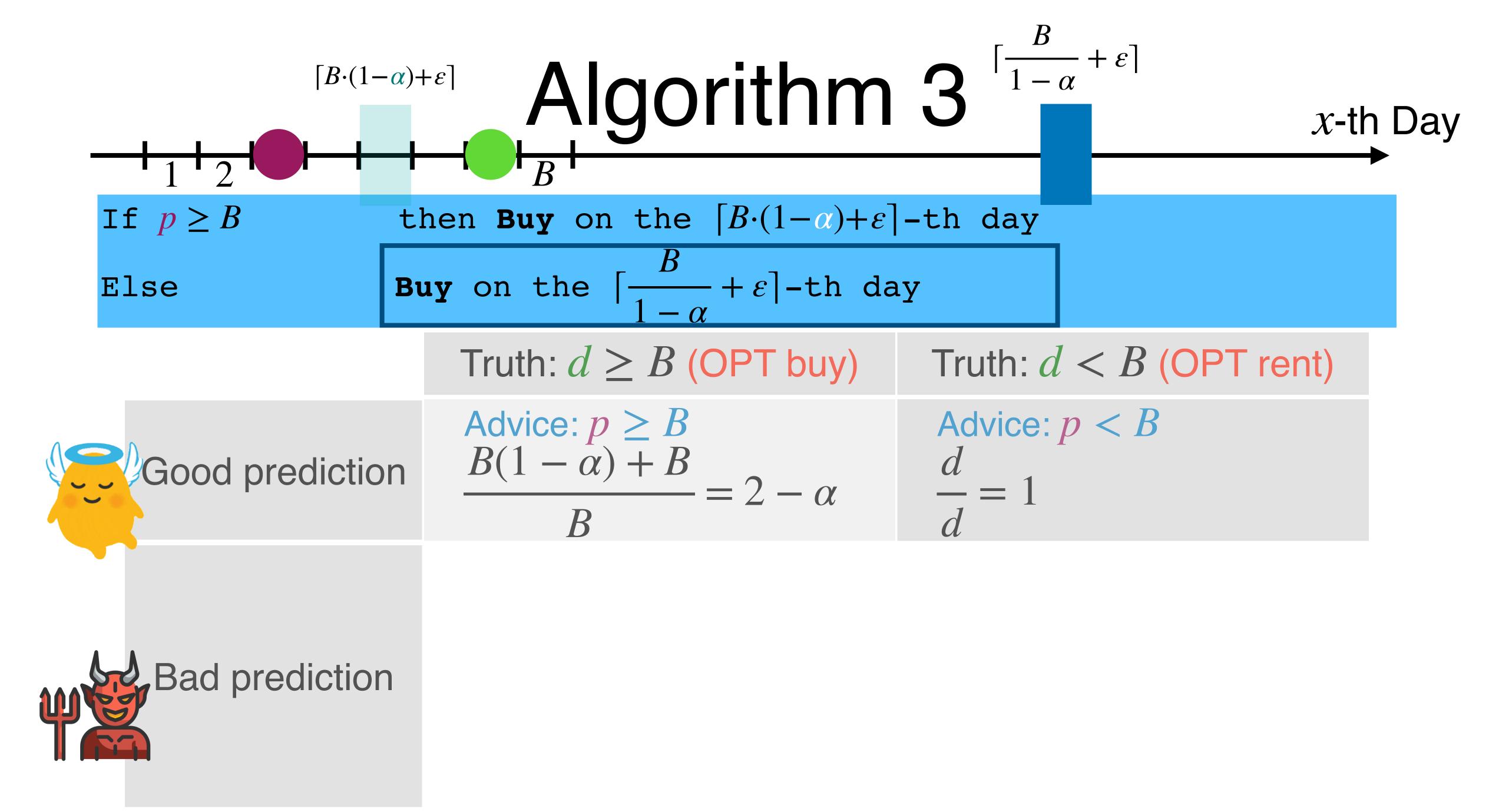
Algorithm 3

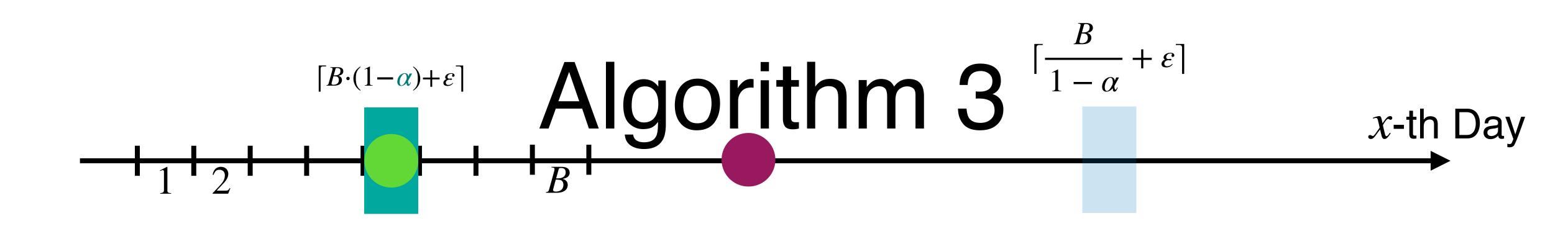


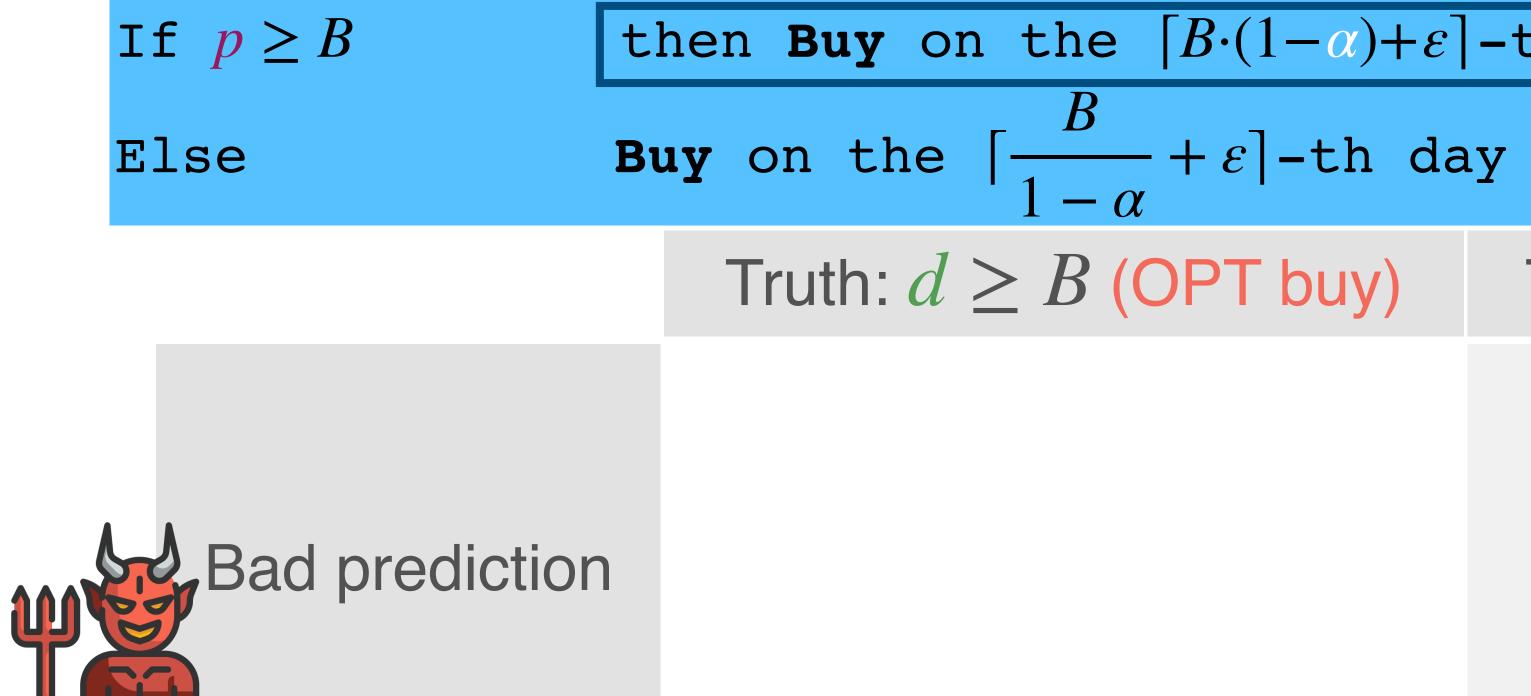
then **Buy** on the $[B \cdot (1-\alpha) + \varepsilon]$ -th day

Truth: $d \ge B$ (OPT buy) Truth: d < B (OPT rent)

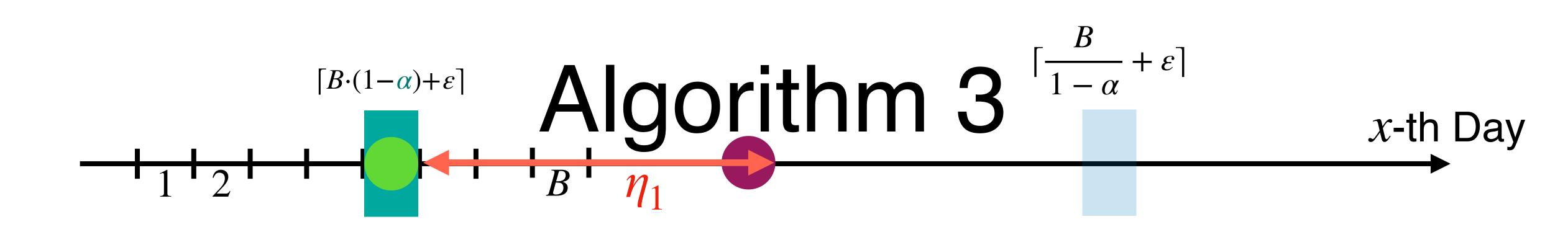


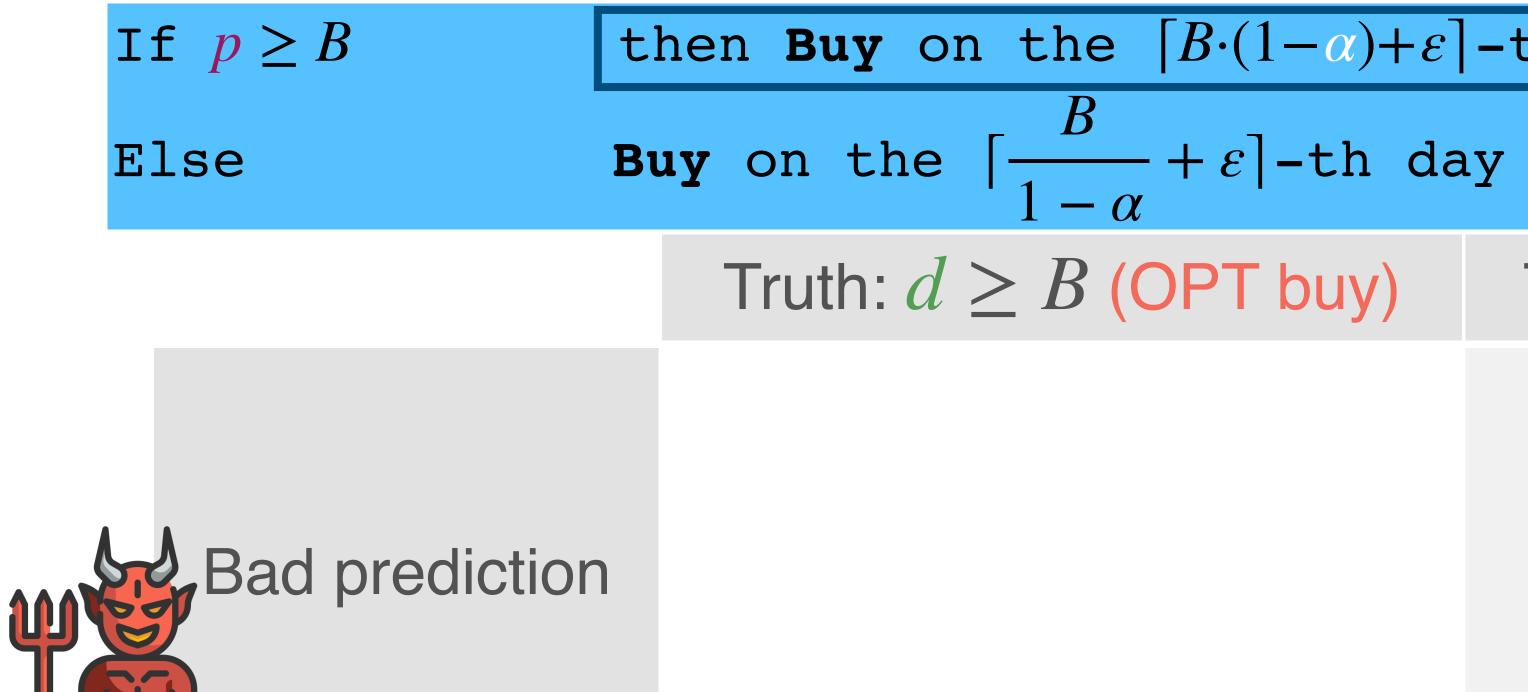




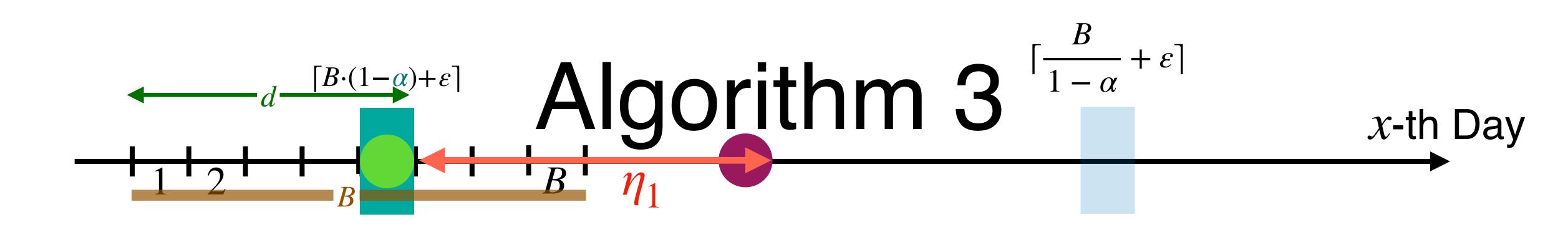


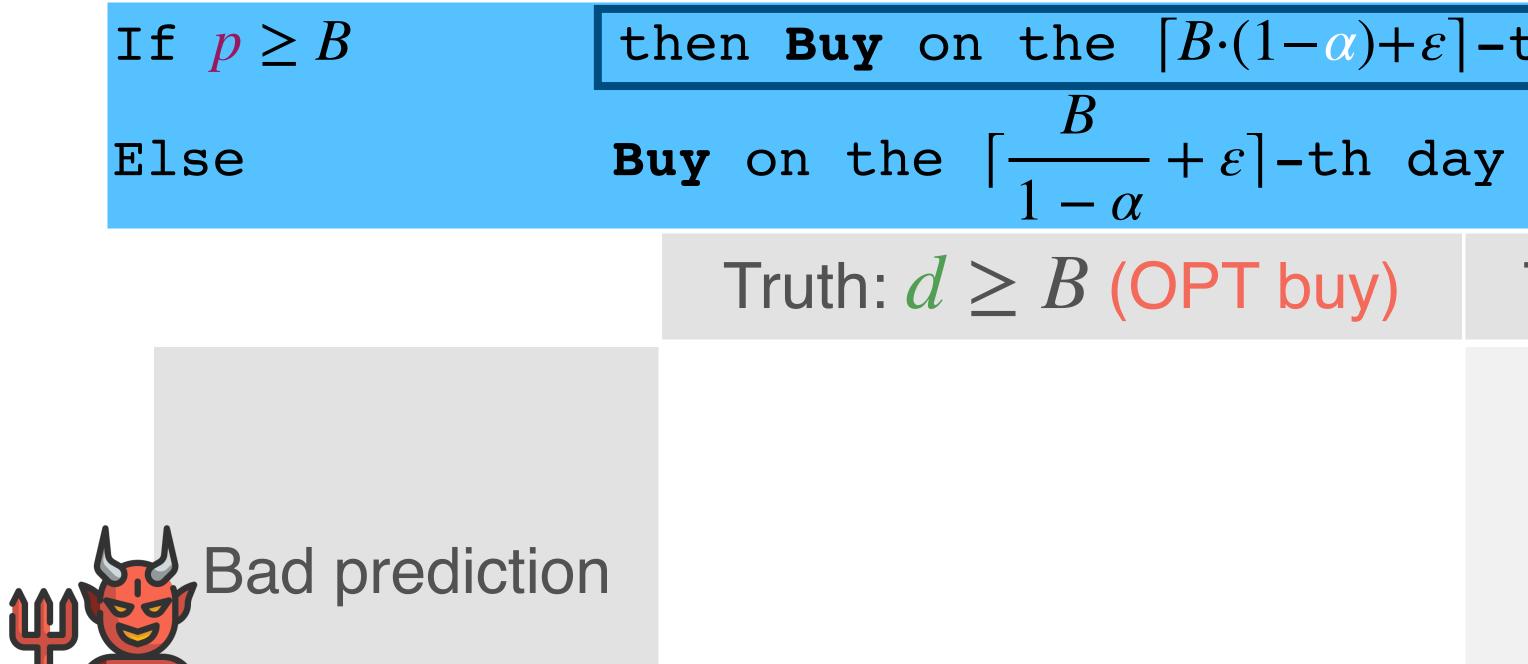
then **Buy** on the $[B \cdot (1-\alpha) + \varepsilon]$ -th day Truth: d < B (OPT rent) Advice: $p \geq B$





then **Buy** on the $[B \cdot (1-\alpha) + \varepsilon]$ -th day Truth: d < B (OPT rent) Advice: $p \ge B \Rightarrow \eta_1 = p - d$

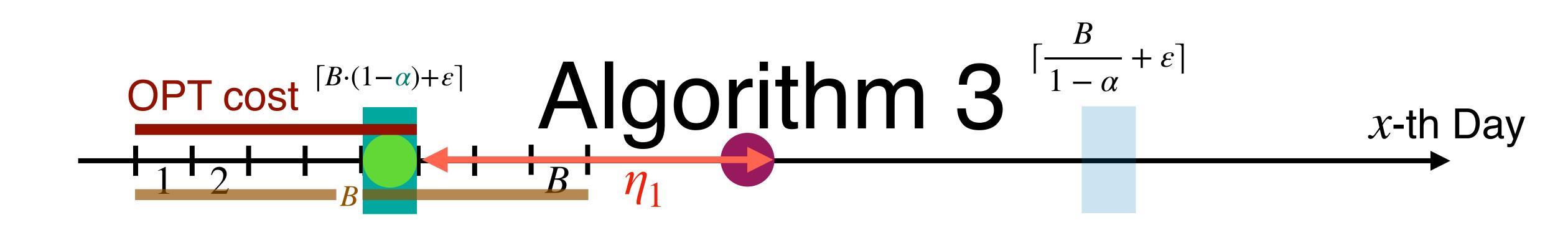


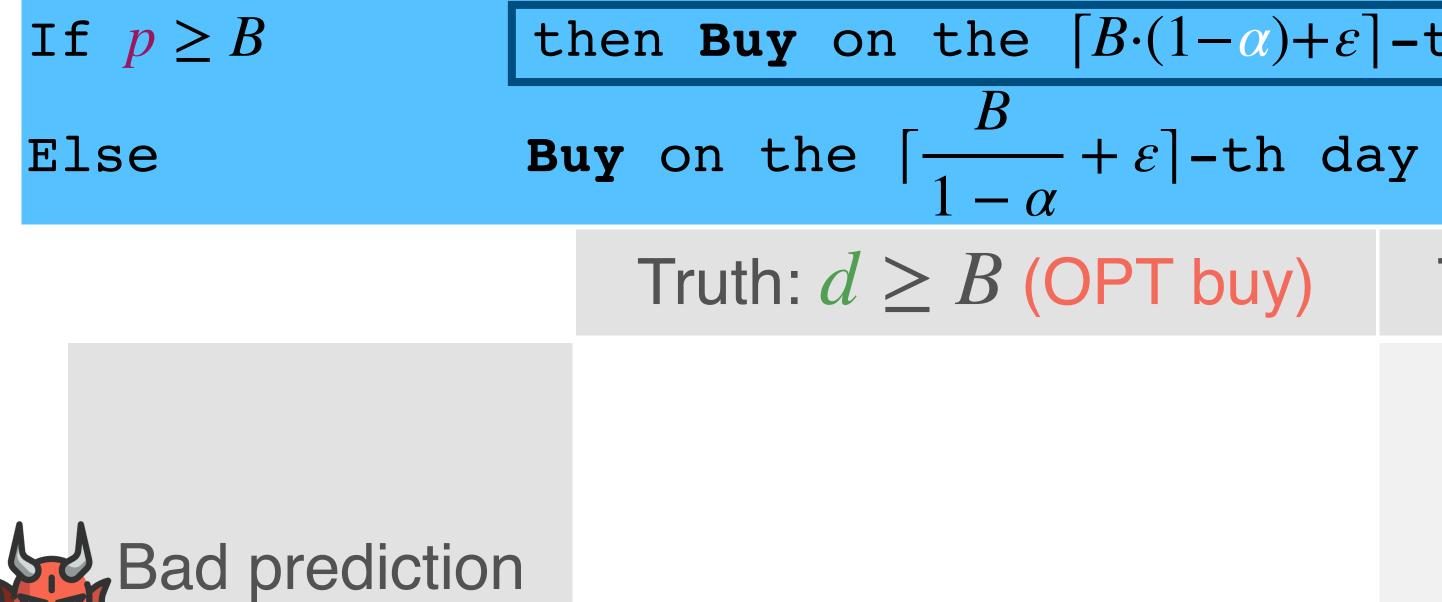


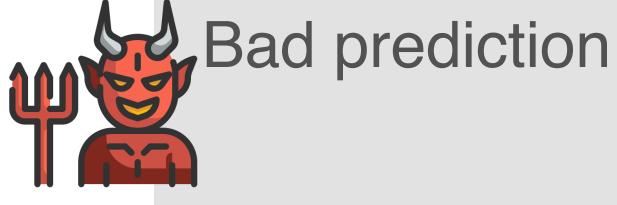
then **Buy** on the $[B \cdot (1-\alpha) + \varepsilon]$ -th day

Truth: d < B (OPT rent)

Advice: $p \ge B \Rightarrow \eta_1 = p - d$ $\Rightarrow B \leq d + \eta_1$



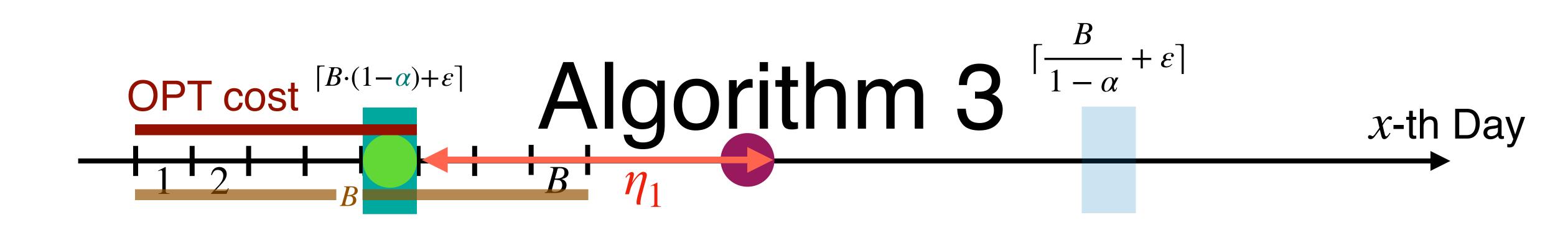


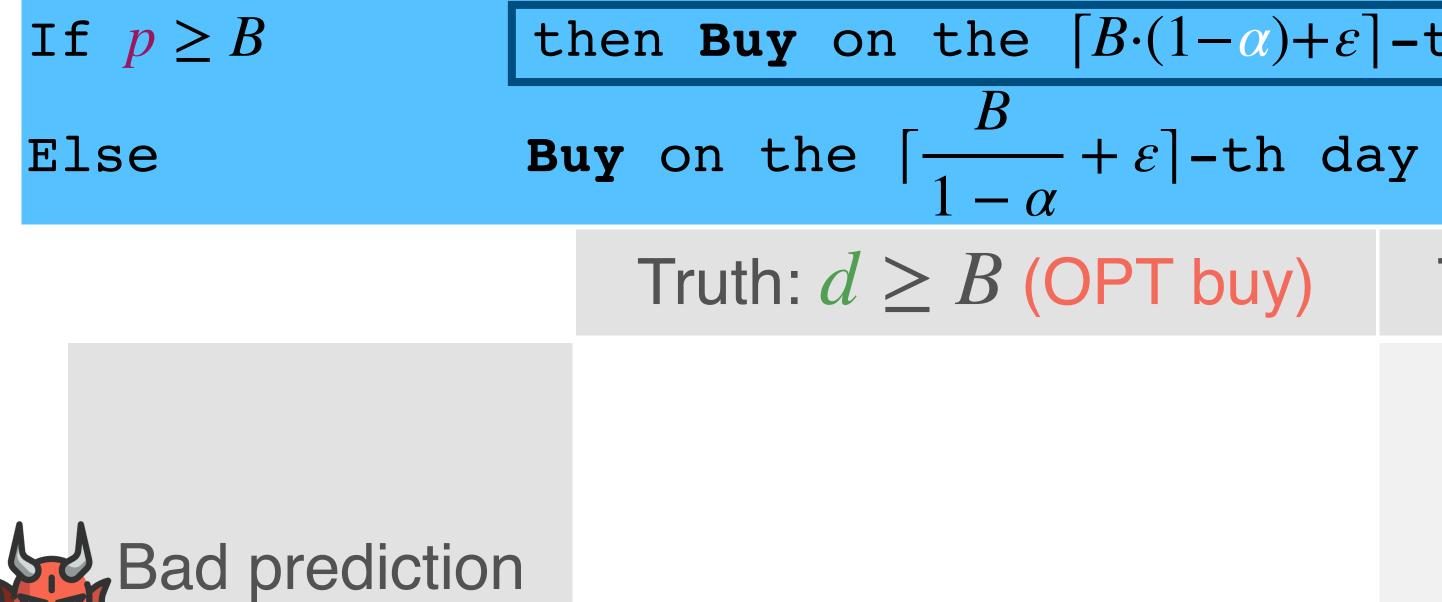


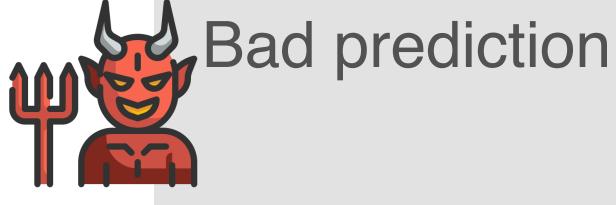
then **Buy** on the $[B \cdot (1-\alpha) + \varepsilon]$ -th day

Truth: d < B (OPT rent)

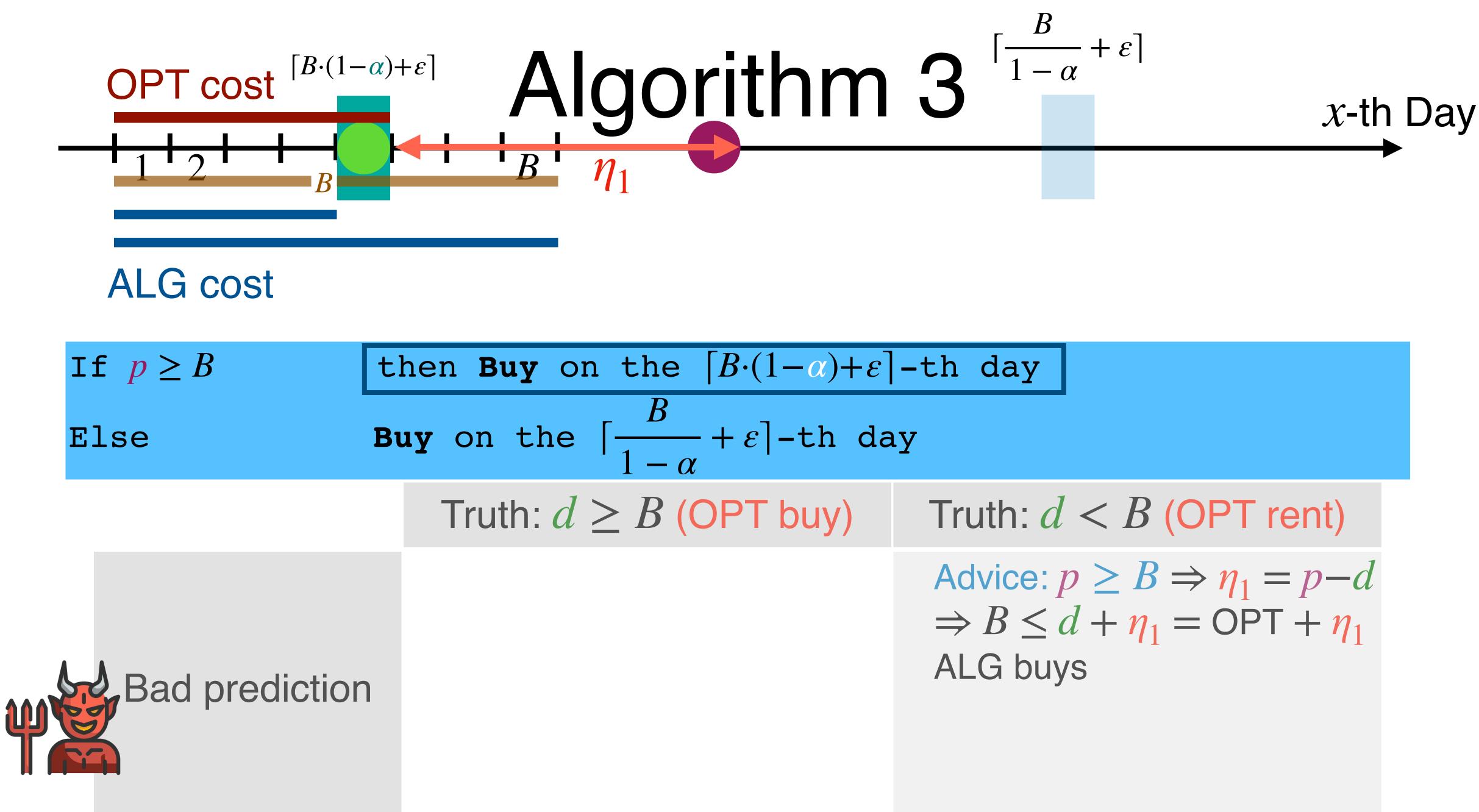
Advice: $p \ge B \Rightarrow \eta_1 = p - d$ $\Rightarrow B \leq d + \eta_1 = OPT + \eta_1$

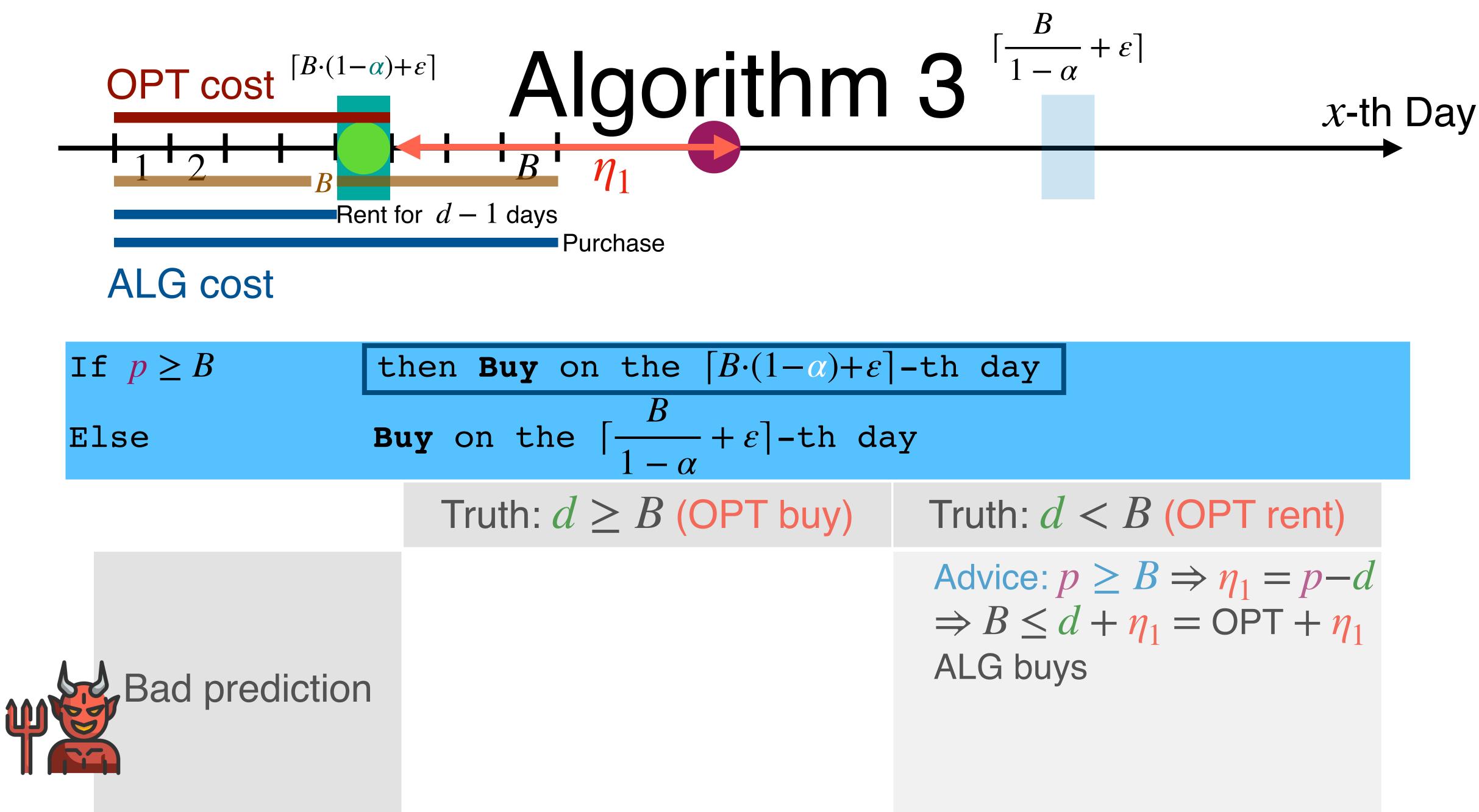


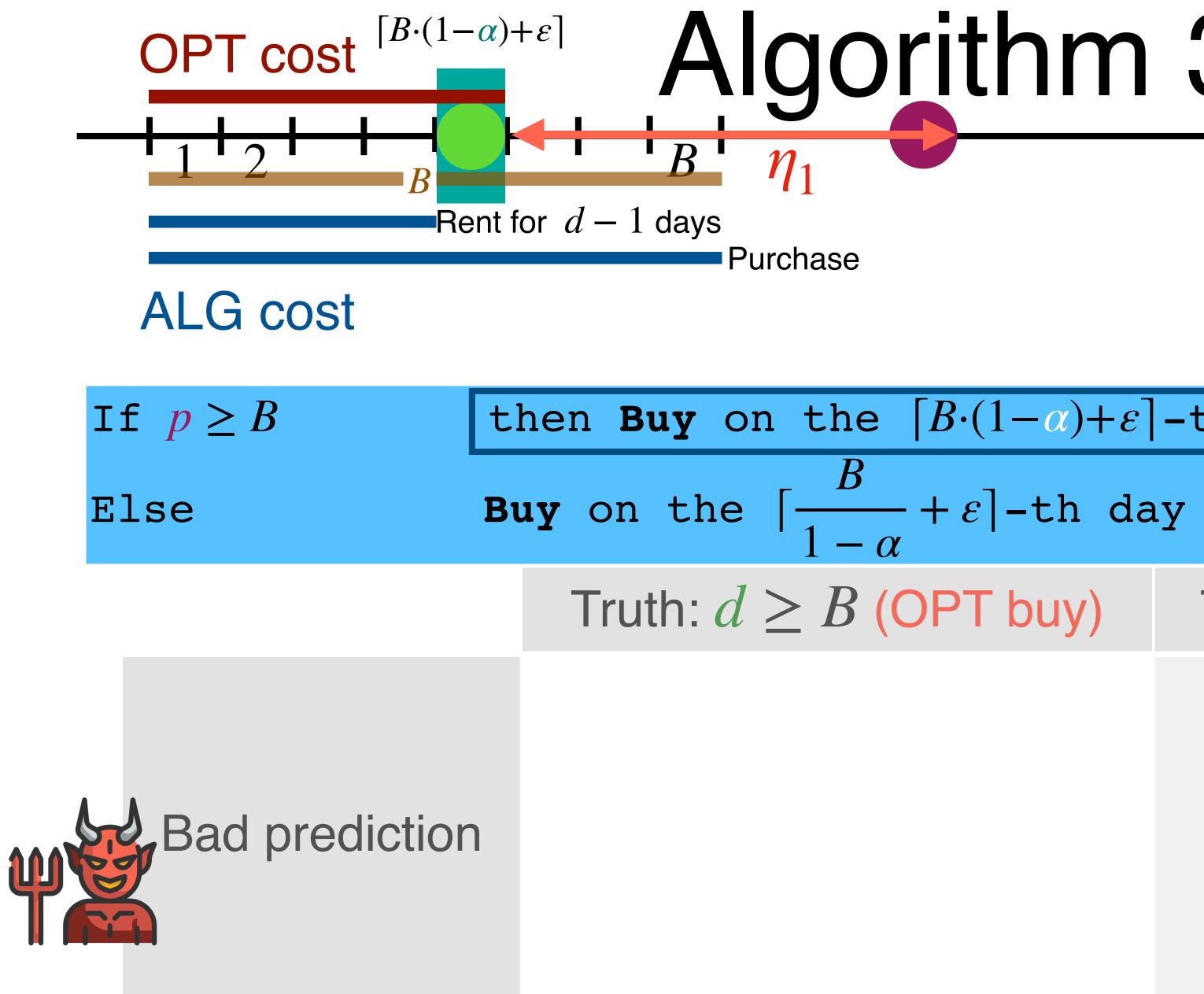




then **Buy** on the $[B \cdot (1-\alpha) + \varepsilon]$ -th day Truth: d < B (OPT rent) Advice: $p \ge B \Rightarrow \eta_1 = p - d$ $\Rightarrow B \leq d + \eta_1 = OPT + \eta_1$ ALG buys







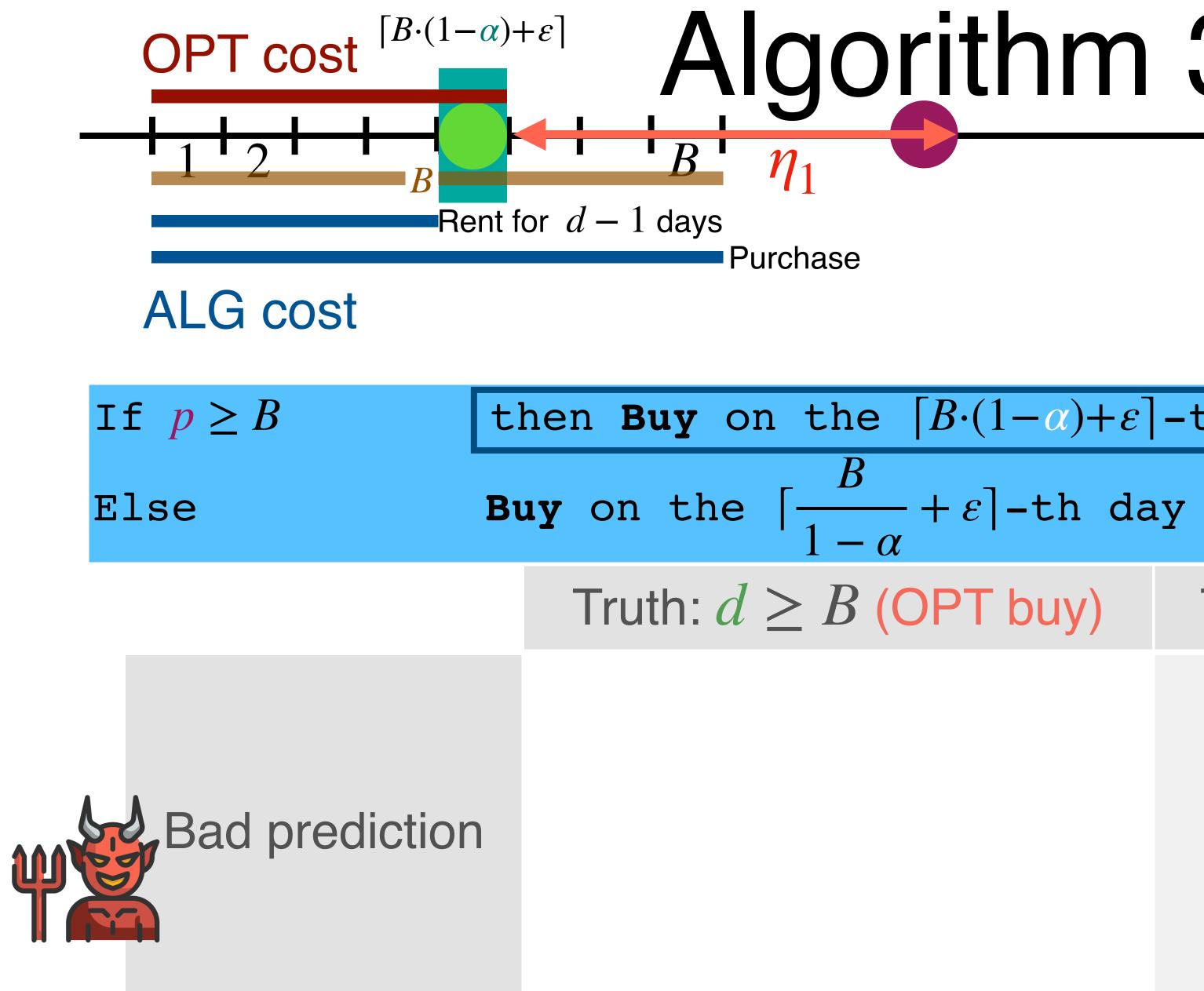
rithm 3
$$\begin{bmatrix} \frac{B}{1-\alpha} + \varepsilon \end{bmatrix}$$
 x-th [

then **Buy** on the $[B \cdot (1-\alpha) + \varepsilon]$ -th day

Truth: d < B (OPT rent)

Advice: $p \ge B \Rightarrow \eta_1 = p - d$ $\Rightarrow B \leq d + \eta_1 = OPT + \eta_1$ ALG buys $\mathsf{ALG} \le B(1 - \alpha) + B$





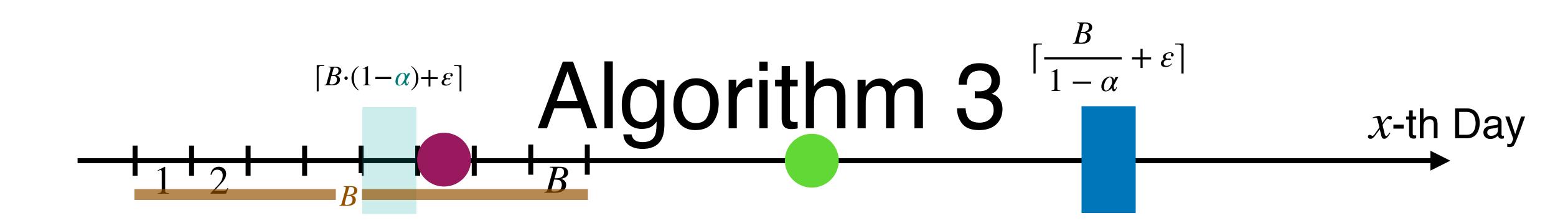
rithm 3
$$\begin{bmatrix} \frac{B}{1-\alpha} + \varepsilon \end{bmatrix}$$
 x-th C

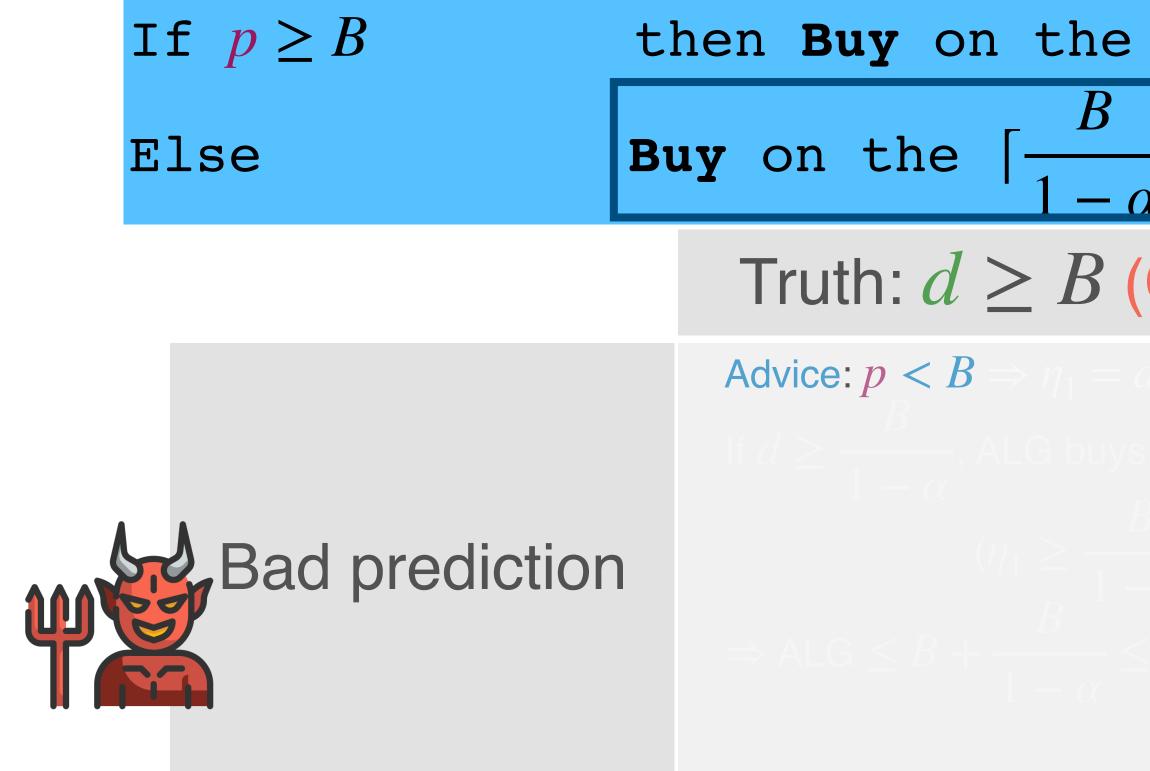
then **Buy** on the $[B \cdot (1-\alpha) + \varepsilon]$ -th day

Truth: d < B (OPT rent)

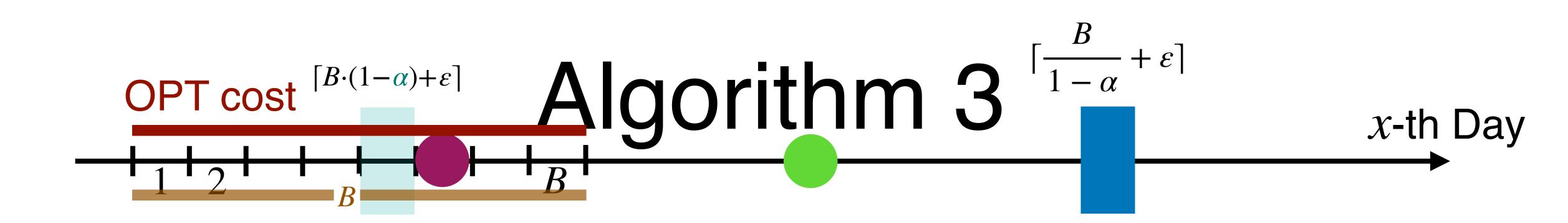
Advice: $p \ge B \Rightarrow \eta_1 = p - d$ $\Rightarrow B \leq d + \eta_1 = OPT + \eta_1$ ALG buys $\mathsf{ALG} \le B(1 - \alpha) + B$ $\leq (2 - \alpha)(\mathsf{OPT} + \eta_1)$

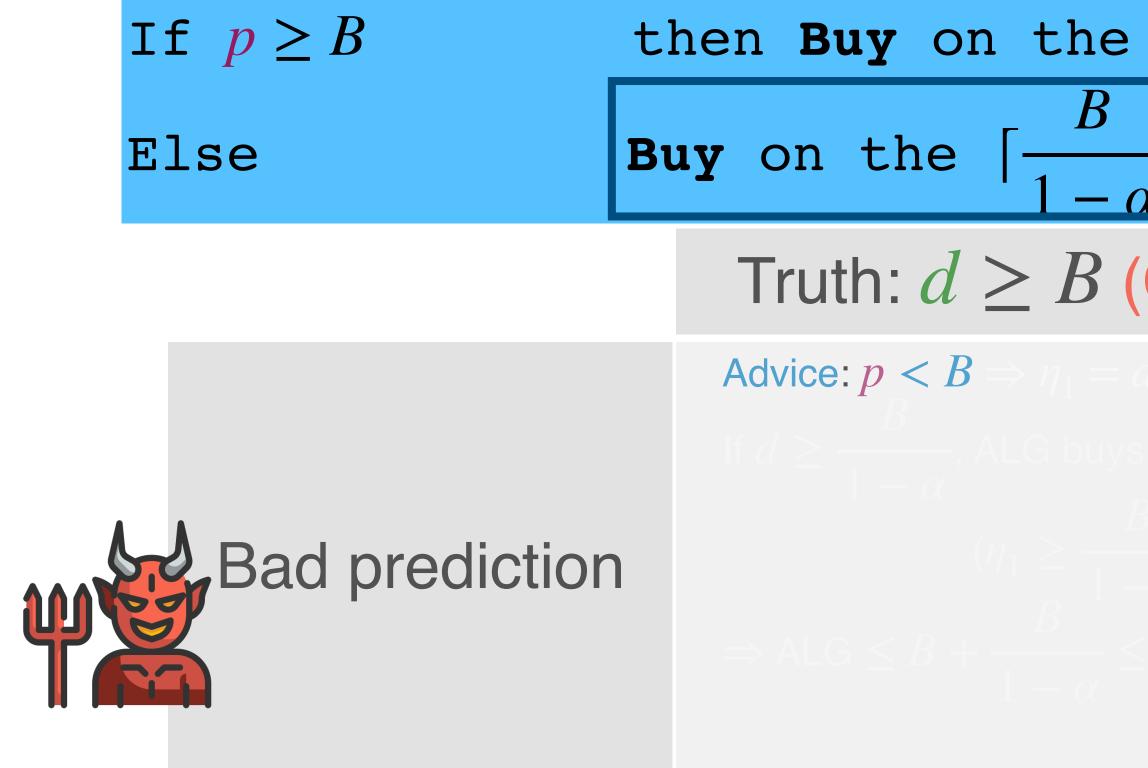




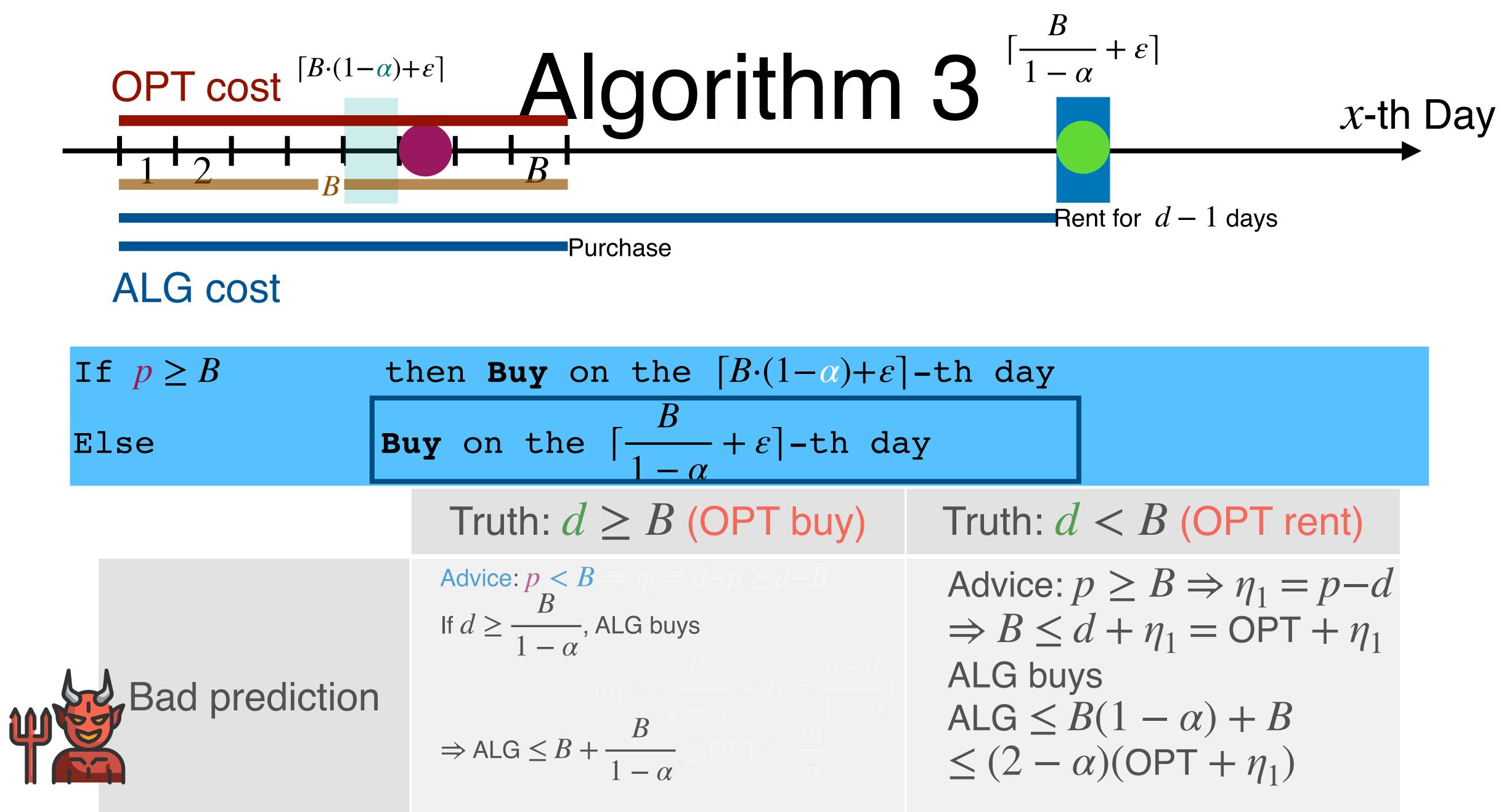


$[B \cdot (1-\alpha) + \varepsilon]$ -th day		
$\frac{1}{\alpha} + \varepsilon - th da$	Ŋ	
(OPT buy)	Truth: $d < B$ (OPT rent)	
	Advice: $p \ge B \Rightarrow \eta_1 = p - d$ $\Rightarrow B \le d + \eta_1 = OPT + \eta_1$ ALG buys ALG $\le B(1 - \alpha) + B$ $\le (2 - \alpha)(OPT + \eta_1)$	

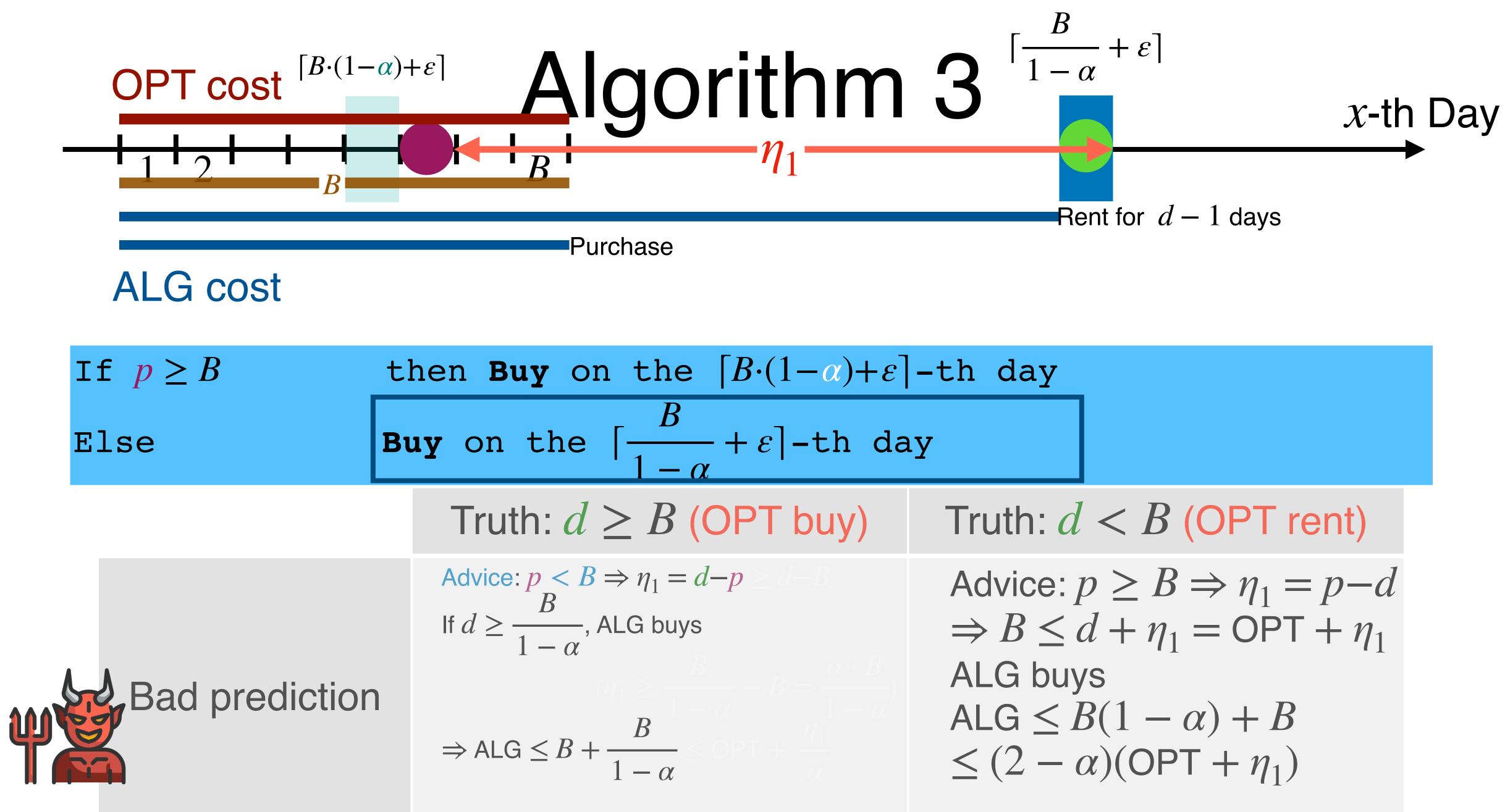




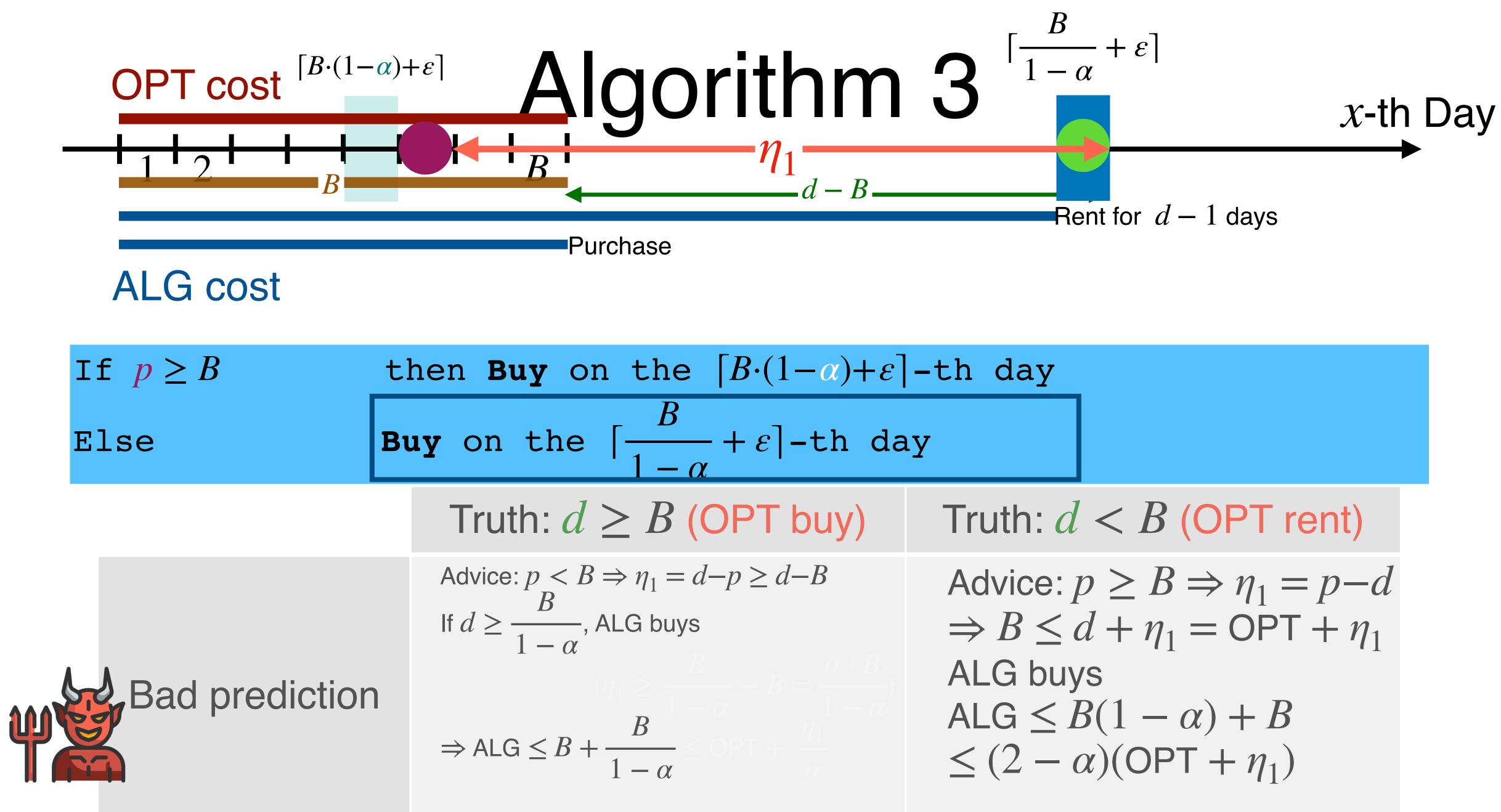
$[B \cdot (1-\alpha) + \varepsilon]$ -th day		
$\frac{1}{\alpha} + \varepsilon - th da$	LY	
(OPT buy)	Truth: $d < B$ (OPT rent)	
	Advice: $p \ge B \Rightarrow \eta_1 = p - d$ $\Rightarrow B \le d + \eta_1 = OPT + \eta_1$ ALG buys ALG $\le B(1 - \alpha) + B$ $\le (2 - \alpha)(OPT + \eta_1)$	



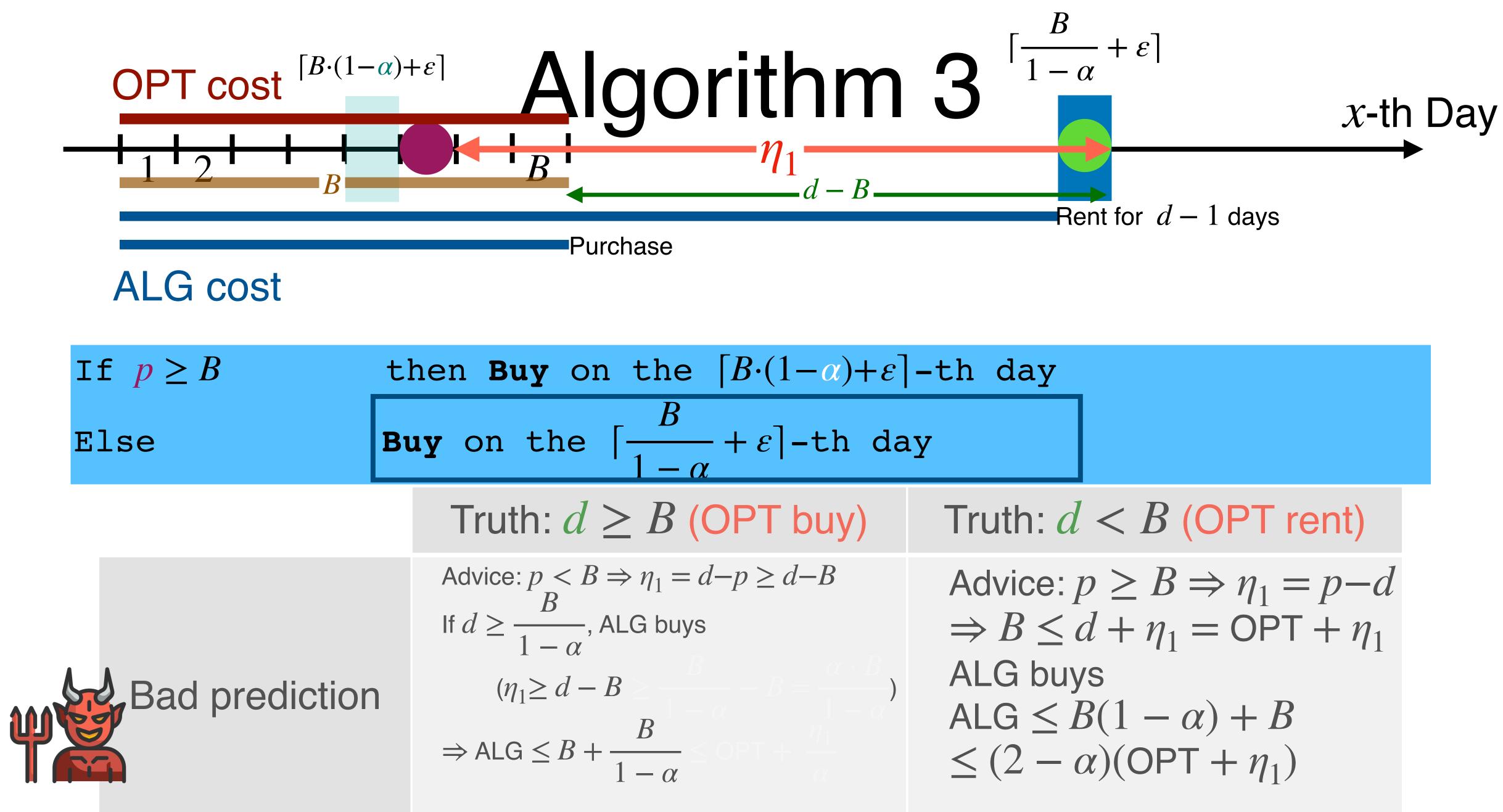
$[B \cdot (1-\alpha) + \varepsilon]$ -th day		
$\alpha + \varepsilon$]-th da	LY	
(OPT buy)	Truth: $d < B$ (OPT rent)	
$d-p \ge d-B$ $B = \alpha \cdot B$ $A = B = (\alpha \cdot B)$ $A = (\alpha \cdot B)$	Advice: $p \ge B \Rightarrow \eta_1 = p - d$ $\Rightarrow B \le d + \eta_1 = OPT + \eta_1$ ALG buys ALG $\le B(1 - \alpha) + B$ $\le (2 - \alpha)(OPT + \eta_1)$	



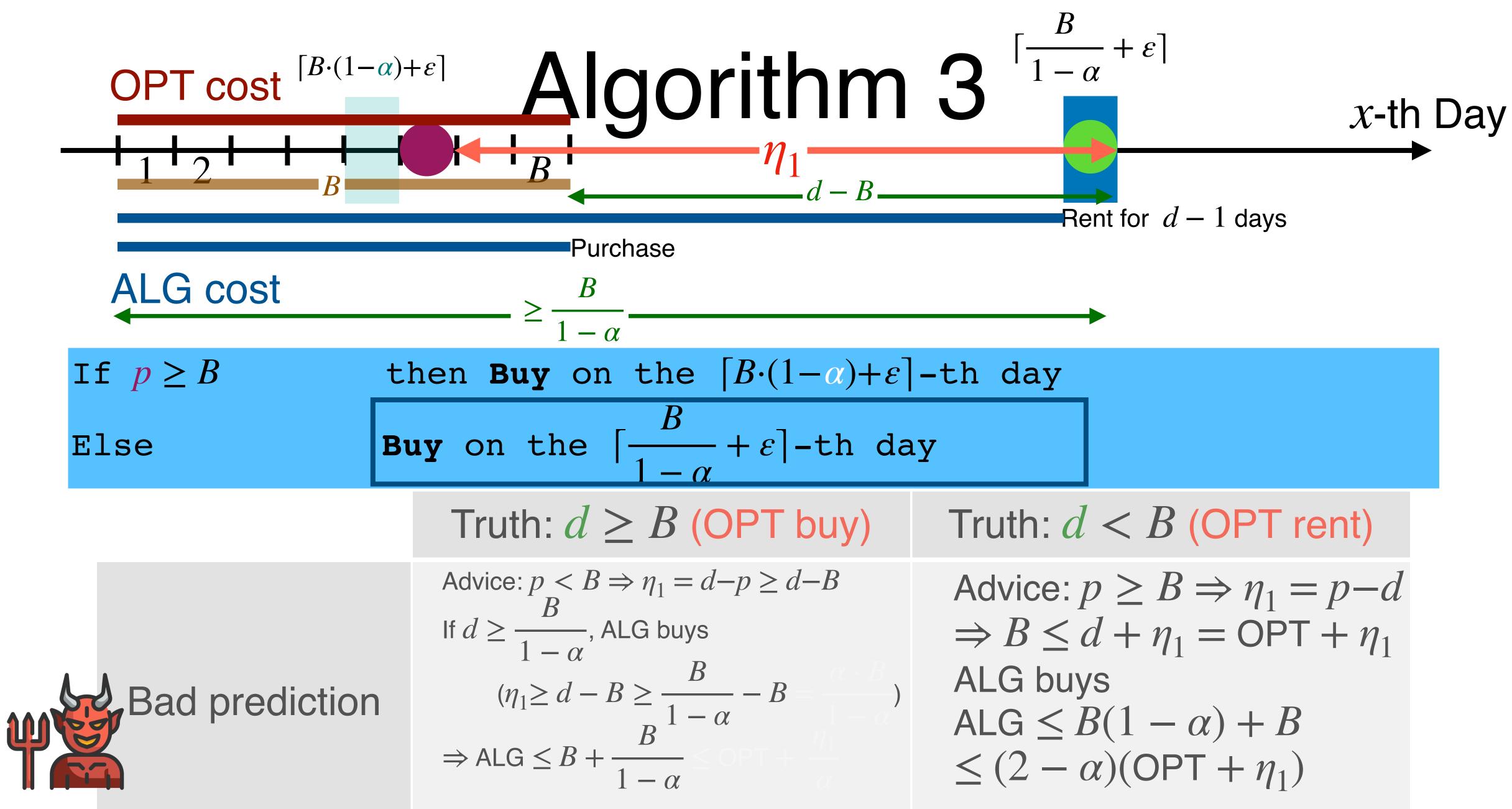
$[B \cdot (1-\alpha) + \varepsilon]$ -th day		
$\alpha + \epsilon$]-th da	чY	
(OPT buy)	Truth: $d < B$ (OPT rent)	
$d-p \ge d-B$ $B = \alpha \cdot B$ $B = \alpha \cdot B$ $A = \beta - \beta = \alpha \cdot B$ $A = \beta - \beta - \alpha \cdot B$ $A = \beta - \alpha \cdot \beta$	Advice: $p \ge B \Rightarrow \eta_1 = p - d$ $\Rightarrow B \le d + \eta_1 = OPT + \eta_1$ ALG buys ALG $\le B(1 - \alpha) + B$ $\le (2 - \alpha)(OPT + \eta_1)$	



$[B \cdot (1-\alpha) + \varepsilon]$ -th day	
$\alpha + \epsilon$]-th da	L.Y
(OPT buy)	Truth: $d < B$ (OPT rent)
$d-p \ge d-B$	Advice: $p \ge B \Rightarrow \eta_1 = p - d$ $\Rightarrow B \le d + \eta_1 = OPT + \eta_1$ ALG buys ALG $\le B(1 - \alpha) + B$ $\le (2 - \alpha)(OPT + \eta_1)$



$[B \cdot (1-\alpha) + \varepsilon]$ -th day	
$\alpha + \varepsilon]$ -th da	LY
(OPT buy)	Truth: $d < B$ (OPT rent)
$d-p \ge d-B$ s $a \cdot B = 0$	$\begin{array}{l} \text{Advice: } p \geq B \Rightarrow \eta_1 = p - d \\ \Rightarrow B \leq d + \eta_1 = \text{OPT} + \eta_1 \\ \text{ALG buys} \\ \text{ALG} \leq B(1 - \alpha) + B \\ \leq (2 - \alpha)(\text{OPT} + \eta_1) \end{array}$

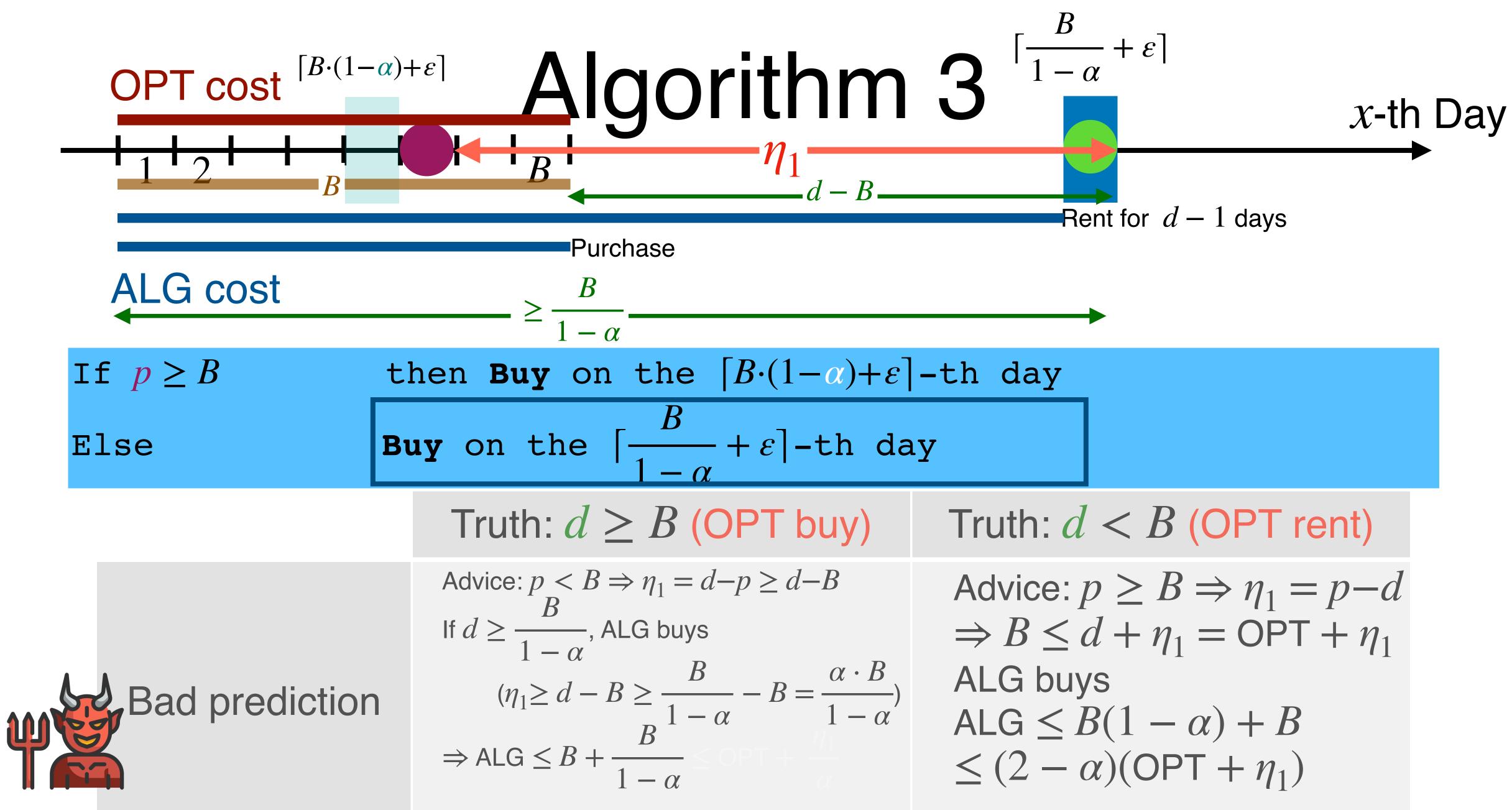


$$[B \cdot (1-\alpha)+\varepsilon] - \text{th day}$$

$$(p+\varepsilon] - \text{th day}$$

$$(p-\varepsilon) = d-B$$

$$(q-\varepsilon) =$$



$$[B \cdot (1-\alpha)+\varepsilon] - \text{th day}$$

$$(-+\varepsilon] - \text{th day}$$
OPT buy)
$$Truth: d < B \text{ (OPT rent)}$$

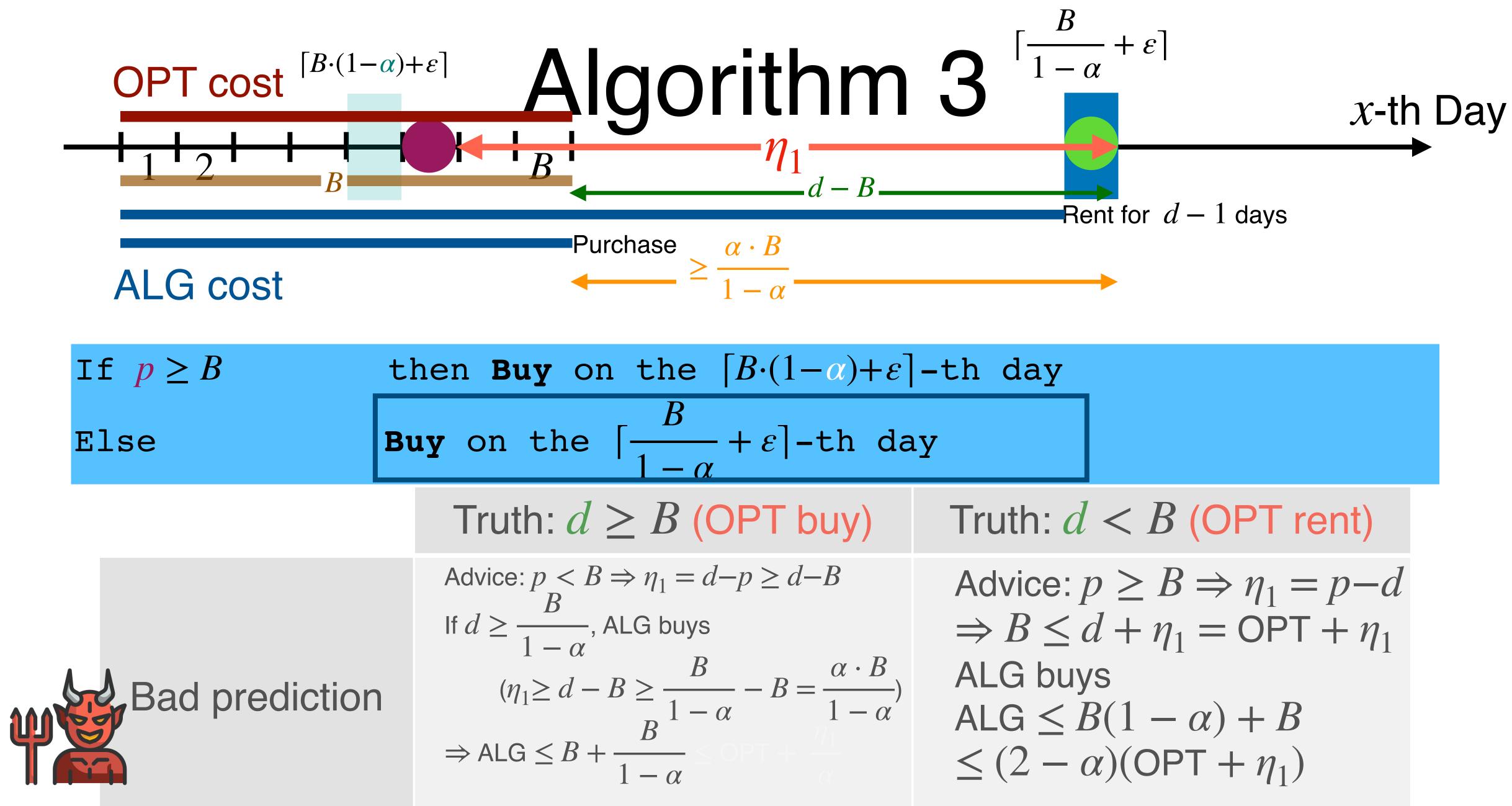
$$Advice: p \ge B \Rightarrow \eta_1 = p - d$$

$$\Rightarrow B \le d + \eta_1 = \text{OPT} + \eta_1$$

$$ALG \text{ buys}$$

$$ALG \le B(1-\alpha) + B$$

$$\le (2-\alpha)(\text{OPT} + \eta_1)$$



$$[B \cdot (1-\alpha)+\varepsilon] - \text{th day}$$

$$(p+\varepsilon] - \text{th day}$$

$$(p+\varepsilon) - \text{th day}$$

$$Truth: d < B \text{ (OPT rent)}$$

$$d-p \ge d-B$$

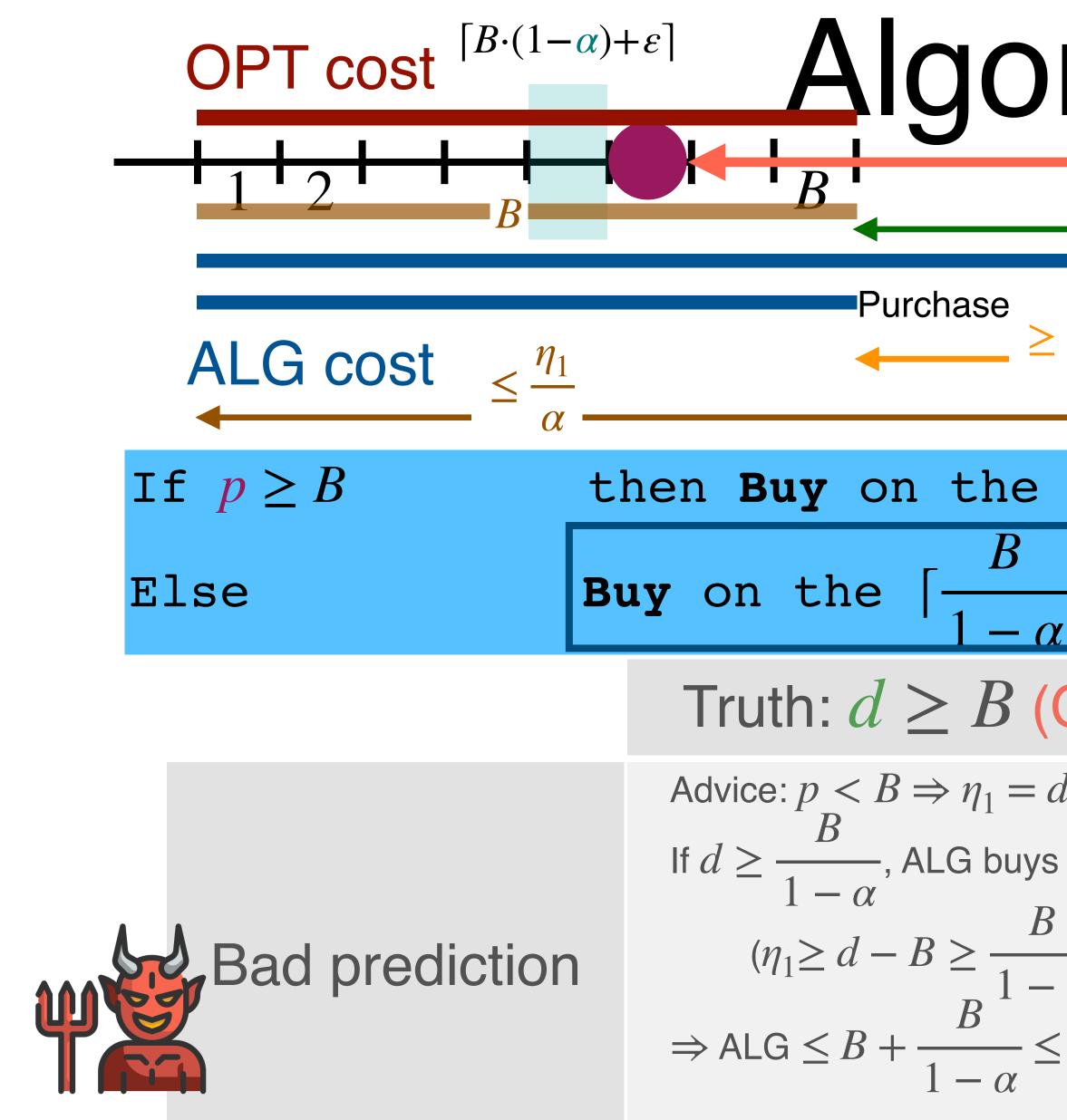
$$Advice: p \ge B \Rightarrow \eta_1 = p-d$$

$$\Rightarrow B \le d + \eta_1 = \text{OPT} + \eta_1$$

$$ALG \text{ buys}$$

$$ALG \le B(1-\alpha) + B$$

$$\le (2-\alpha)(\text{OPT} + \eta_1)$$



rithm 3

$$\begin{bmatrix} B \\ 1-\alpha \end{bmatrix}$$
Rent for $d-1$ days

$$\begin{bmatrix} a \cdot B \\ 1-\alpha \end{bmatrix}$$
Rent for $d-1$ days

$$\begin{bmatrix} B \cdot (1-\alpha) + \varepsilon \end{bmatrix}$$
-th day

$$\begin{bmatrix} B \cdot (1-\alpha) + \varepsilon \end{bmatrix}$$
-th day

$$\begin{bmatrix} B \cdot (1-\alpha) + \varepsilon \end{bmatrix}$$
-th day

$$\begin{bmatrix} B \cdot (1-\alpha) + \varepsilon \end{bmatrix}$$
-th day

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-th day

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-th day

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-th day

$$\begin{bmatrix} B \cdot (1-\alpha) + \varepsilon \end{bmatrix}$$
-th day

$$\begin{bmatrix} B \cdot (1-\alpha) + \varepsilon \end{bmatrix}$$
-th day

$$\begin{bmatrix} A \cdot B \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} A \cdot B \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} A \cdot B \\ 0 \end{bmatrix}$$

$$Advice: p \ge B \Rightarrow \eta_1 = p - d \\ \Rightarrow B \le d + \eta_1 = 0PT + \eta_1$$

$$ALG buys$$

$$ALG \le B(1-\alpha) + B \\ \le (2-\alpha)(0PT + \eta_1)$$



- We can use the absolute error measure of the prediction to represent the robustness
 - More specifically, ALG3's cost is a function of 1) the error measure, 2) the trust parameter, and 3) OPT cost

What Happened

→consistency

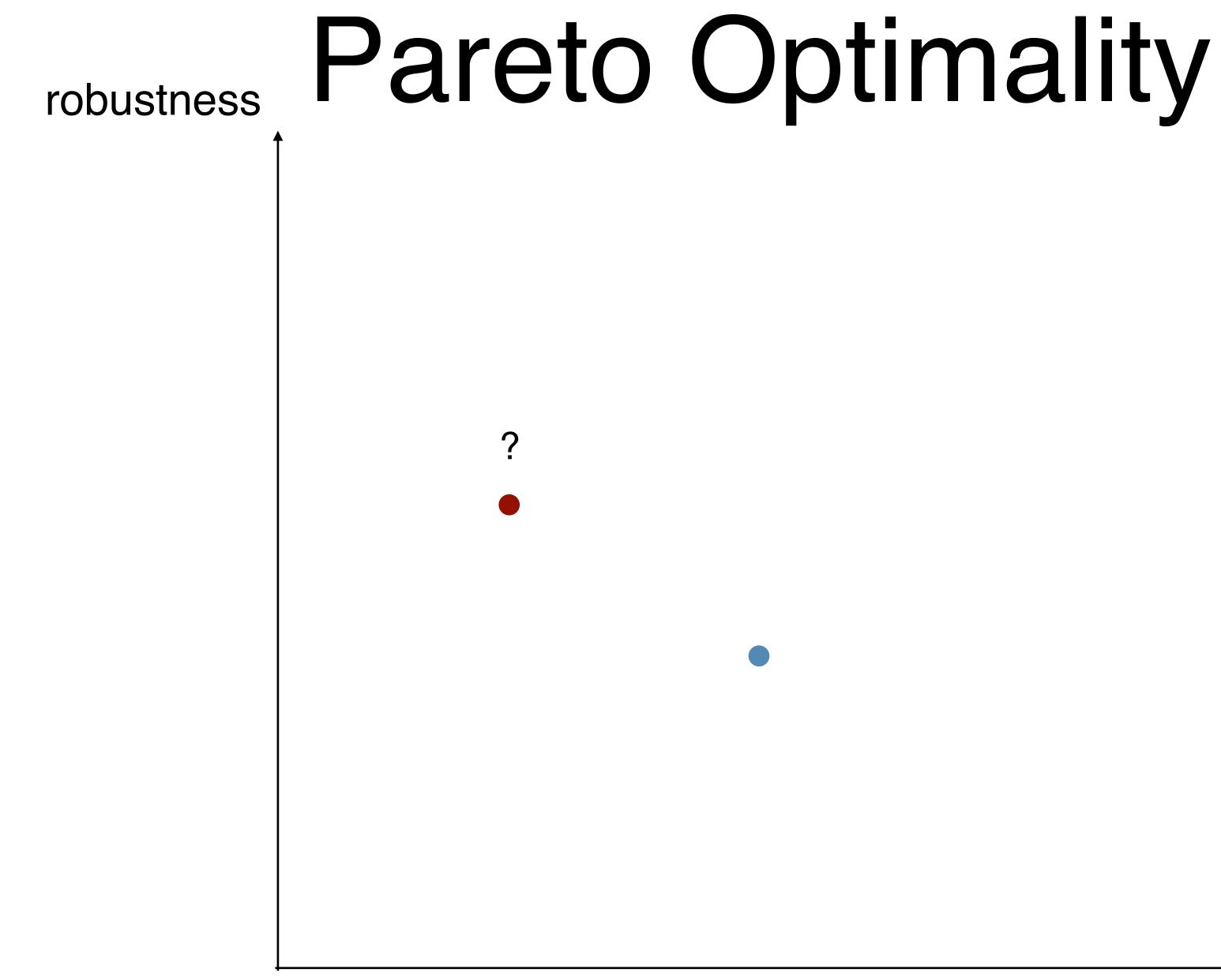
→ consistency

better

→ consistency

worse

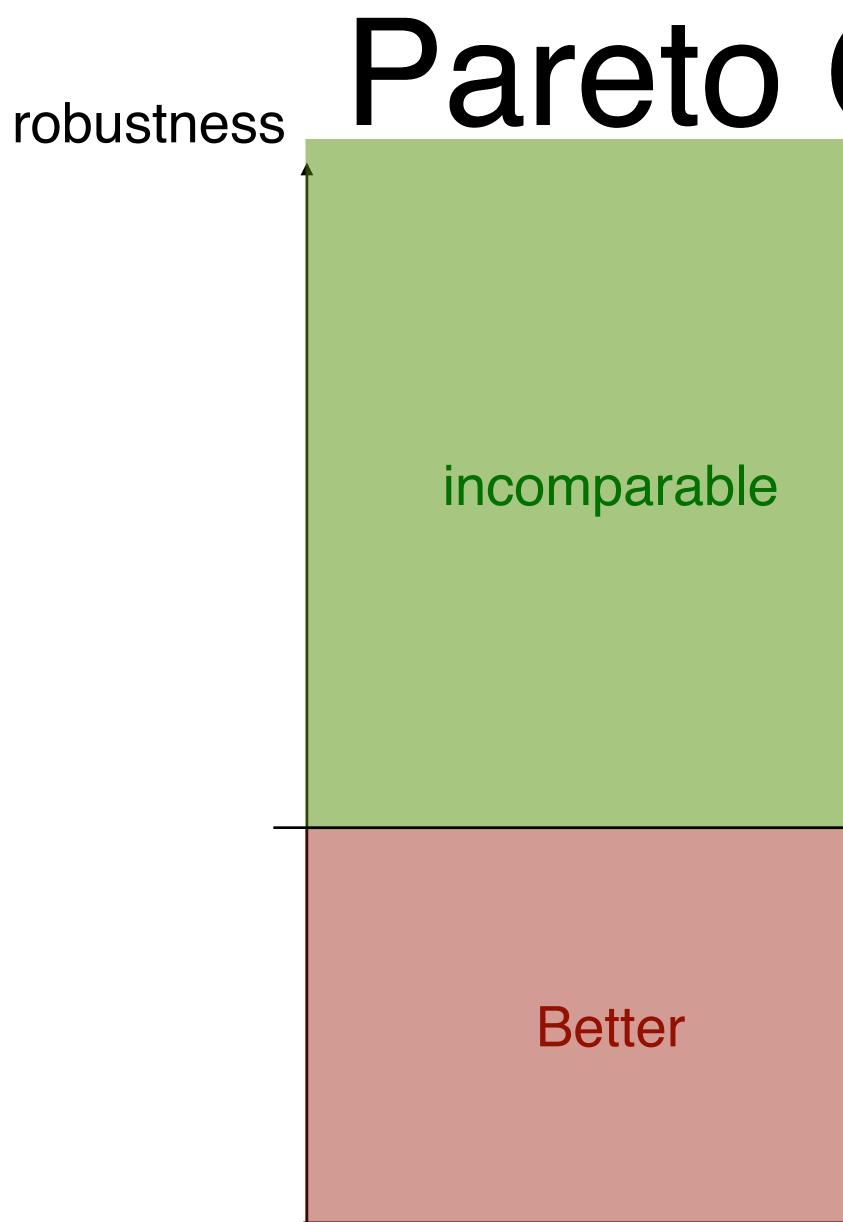
→ consistency



→consistency

→consistency

?



Pareto Optimality

worse

incomparable

→ consistency

- - ALG1 is better than ALG2 if $\alpha_1 \leq \alpha_2$ and $\beta_1 \leq \beta_2$
 - ALG is Pareto optimal if for all ALG', $\alpha \leq \alpha' \text{ or } \beta \leq \beta'$

What Happened

• In a bi-criteria optimization problem (minimizing criteria α and β),

Ski Rental Algorithm 2

SKI-Rental with prediction (p, k)If $p \ge B$ Keep renting until the *k*-th of the else (p < B)Keep renting until the *B*-th of the else (p < B)

Truth:
$$d \ge B$$
 (

Advice:
$$p \ge B$$

 $(k-1) + B$

B

Advice: p < B(B - 1) + B

B



Good prediction Consistency

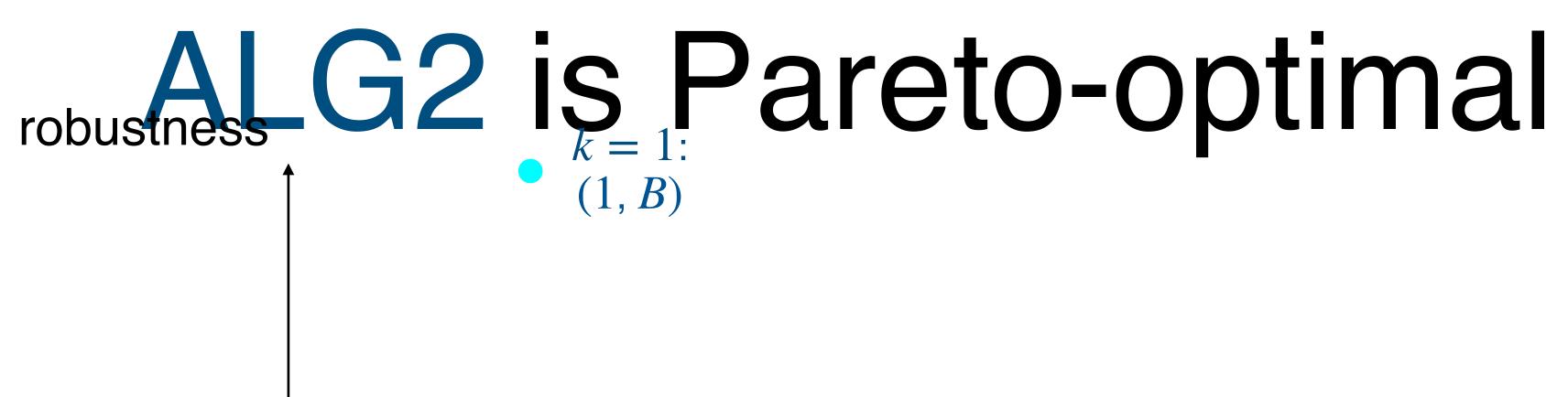
day	// k is our "trust parameter"
day	d: Actual number of skiing days
(OPT buy)	Truth: $d < B$ (OPT rent)
$= 1 + \frac{k-1}{B}$	Advice: $p < B$ $\frac{d}{d} = 1$
$\frac{B}{B} = 2 + \frac{1}{B}$	$\frac{\text{Advice: } p \ge B}{(k-1)+B} = 1 + \frac{B-1}{k}$



robustness G2 is Pareto-optimal

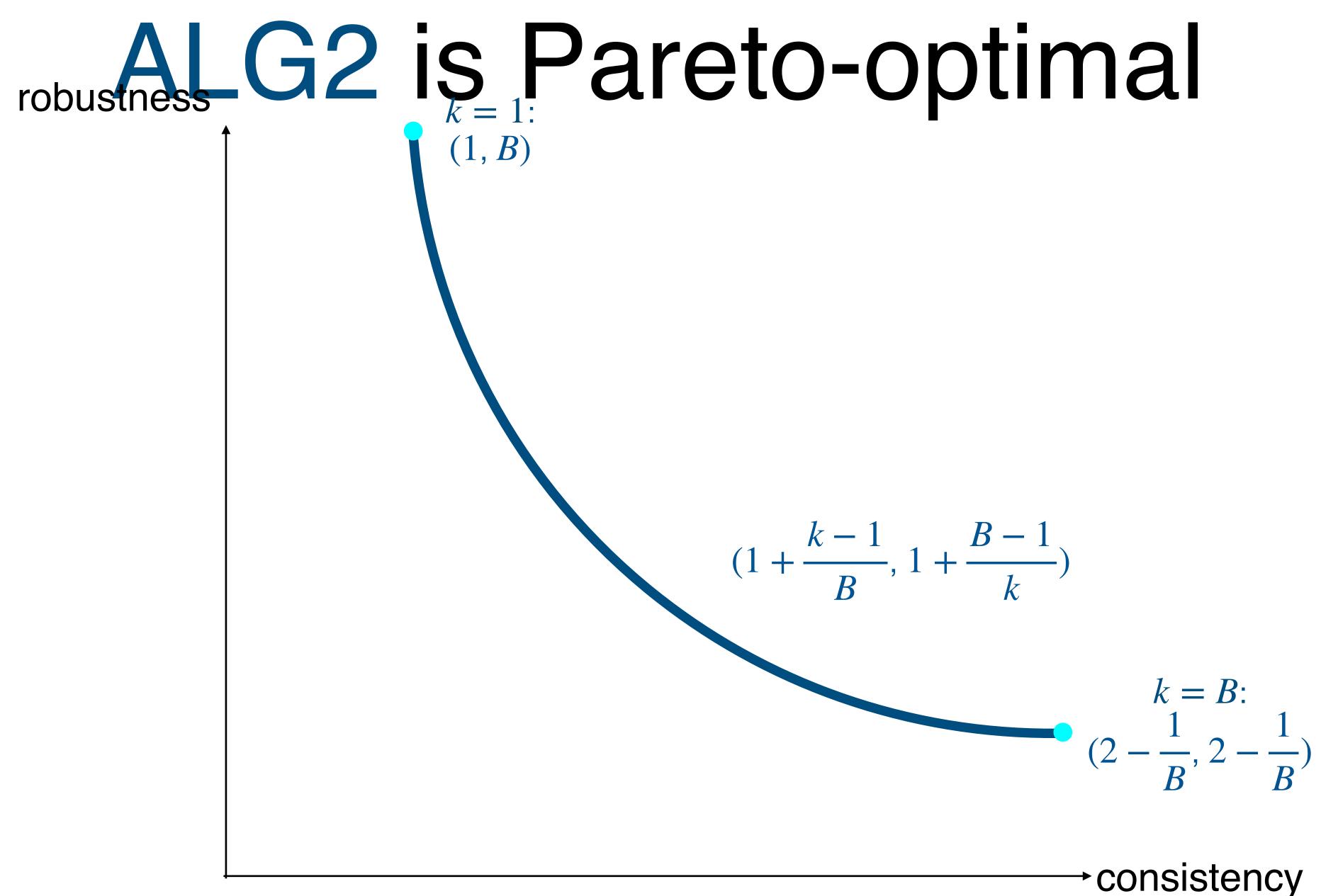
$$k = B$$
:
(2 $-\frac{1}{B}$, 2 $-\frac{1}{B}$)

→consistency



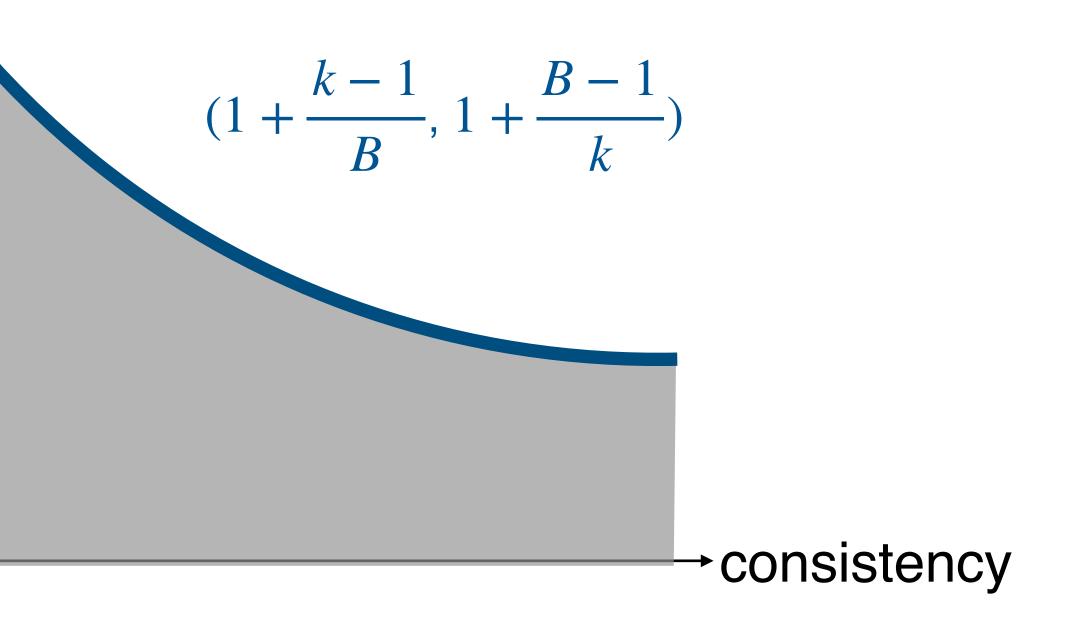
$$k = B$$
:
(2 $-\frac{1}{B}$, 2 $-\frac{1}{B}$)

→consistency

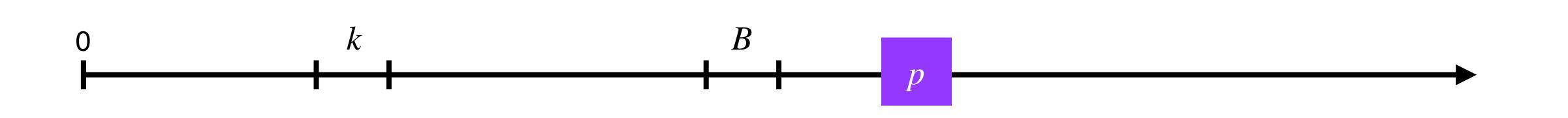


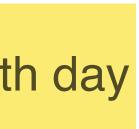
robustness G2 is Pareto-optimal

(We want to show) nothing here



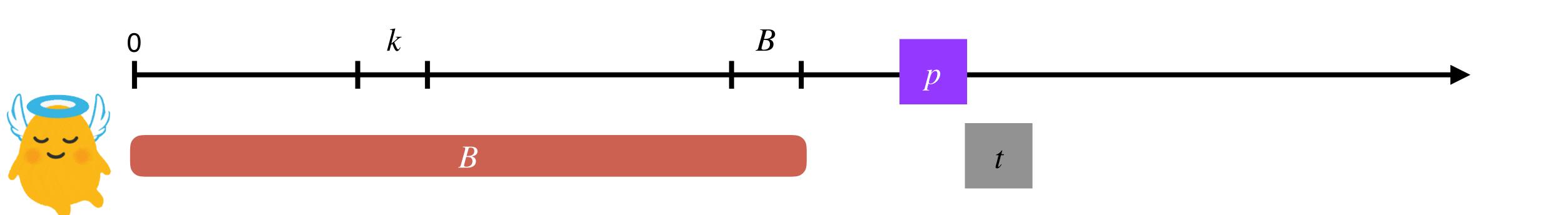
Any algorithm A with consistency of at most $1 + \frac{k-1}{B}$ against prediction $p \ge B$ must buy the ski before the k-th day

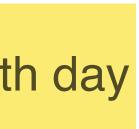




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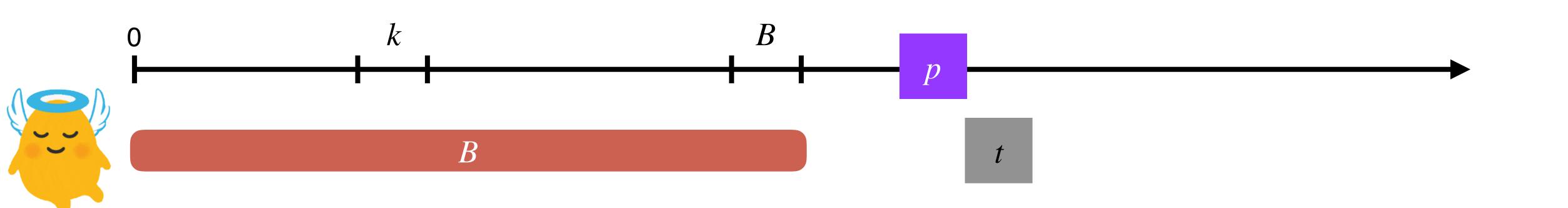
OPT = B



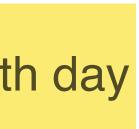


Any algorithm A with consistency of at most $1 + \frac{k-1}{B}$ against prediction $p \ge B$ must buy the ski before the k-th day

OPT = B

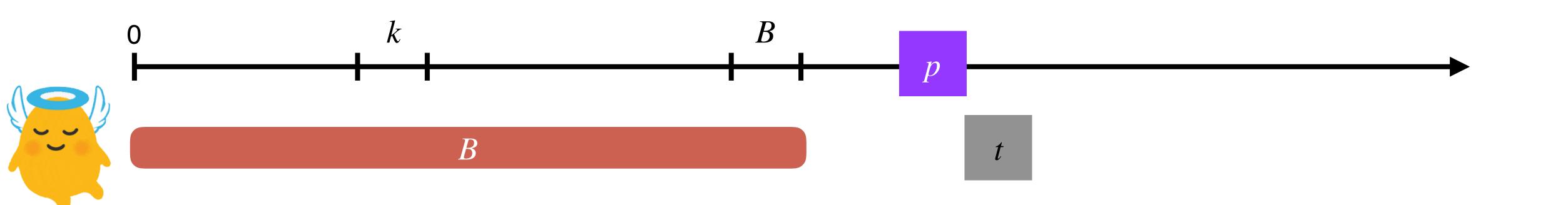


$$\mathsf{ALG} \le (1 + \frac{k-1}{B}) \cdot B$$

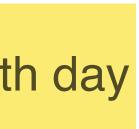


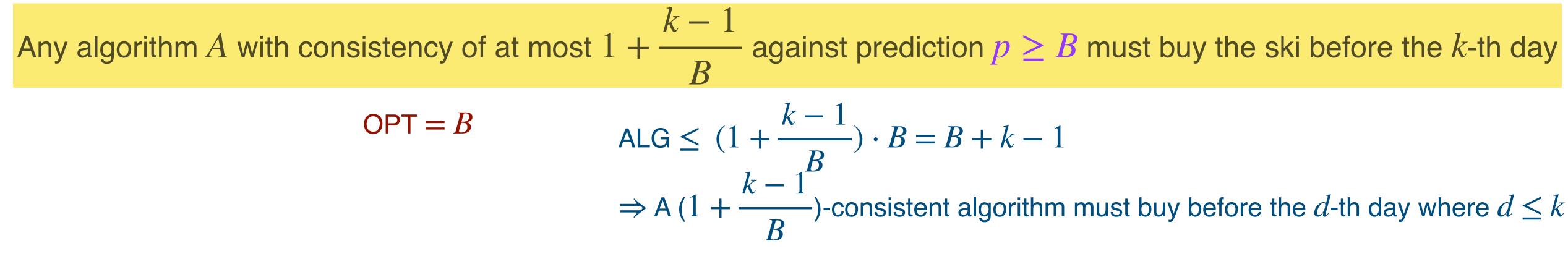
Any algorithm A with consistency of at most $1 + \frac{k-1}{B}$ against prediction $p \ge B$ must buy the ski before the k-th day

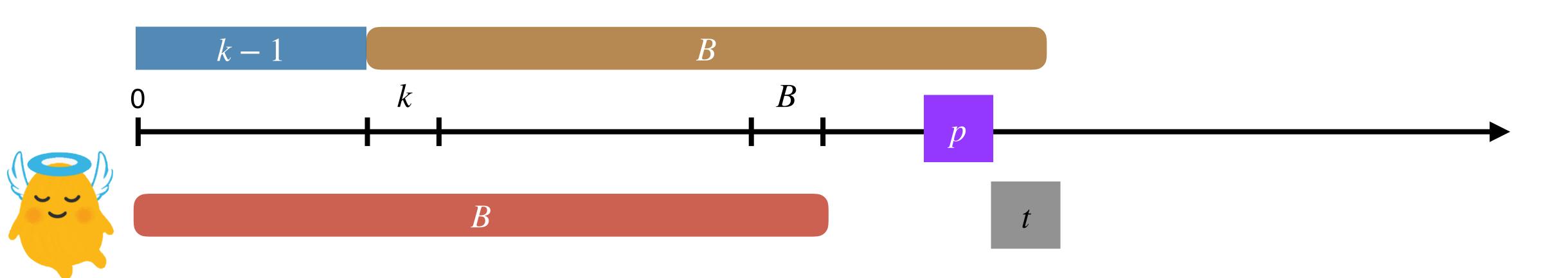
OPT = B



$$\mathsf{ALG} \le (1 + \frac{k-1}{B}) \cdot B = B + k - 1$$

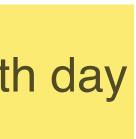




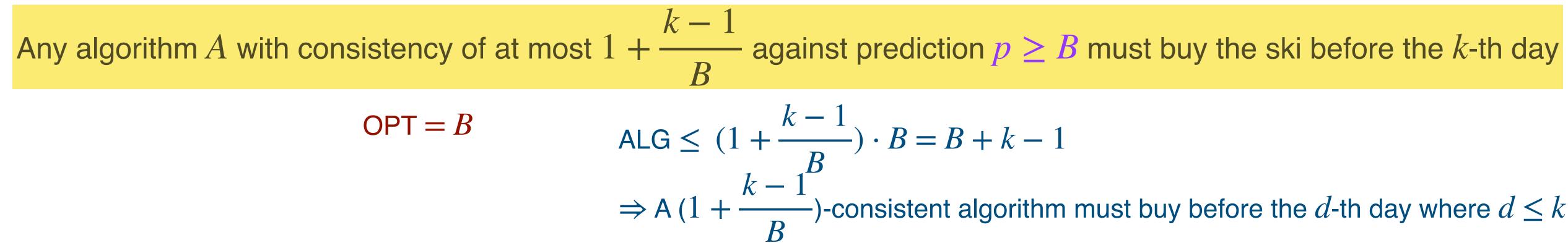


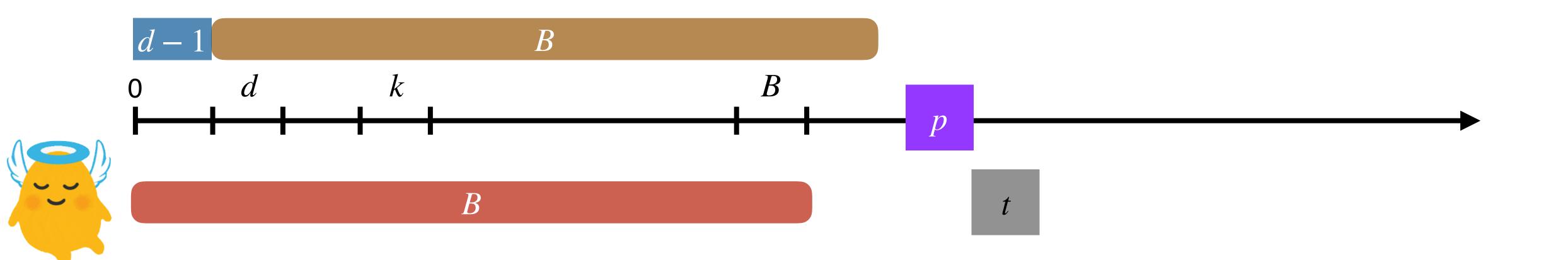
$$+\frac{k-1}{B}) \cdot B = B + k - 1$$

$$\frac{k-1}{B}$$
)-consistent algorithm must buy before the *d*-th day where

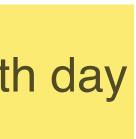




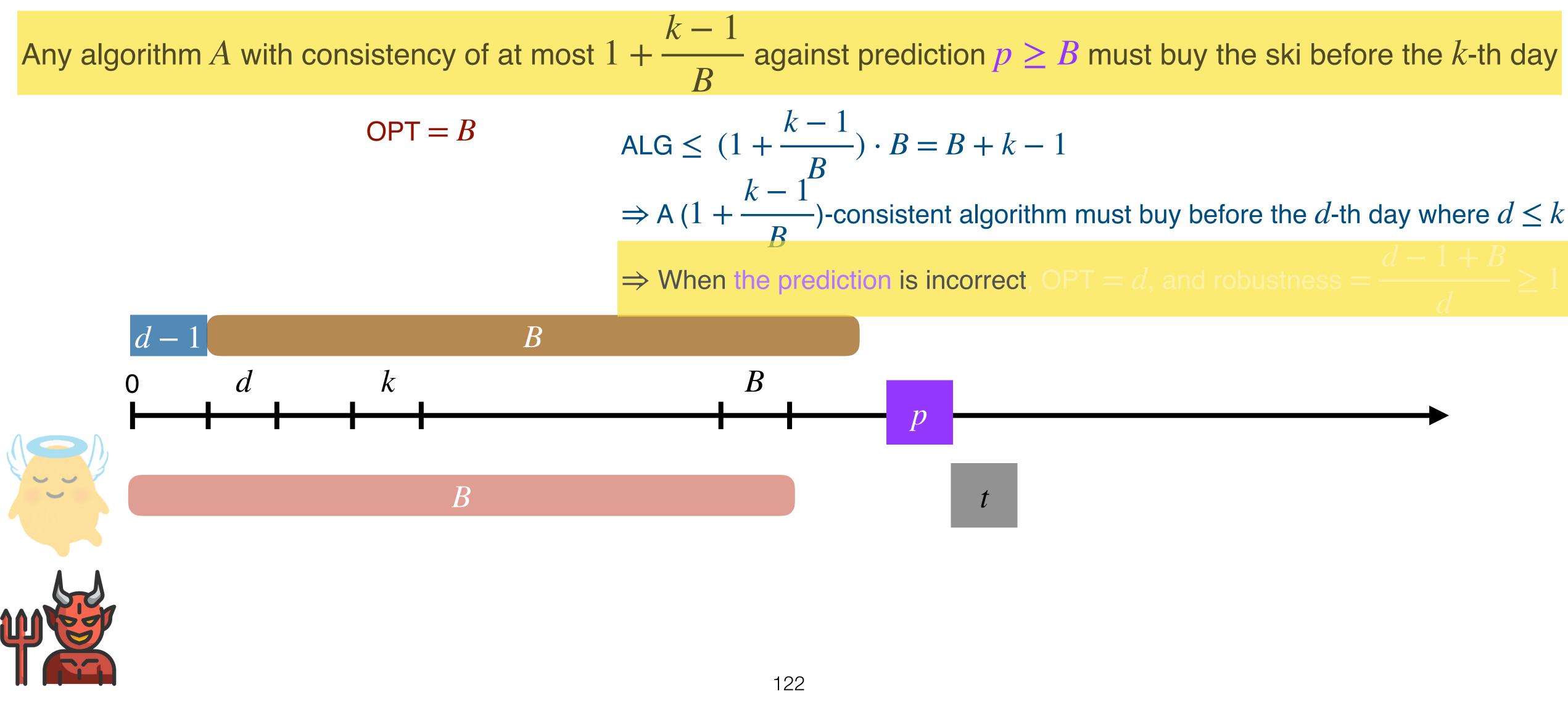




 $ALG \le (1 + \frac{k-1}{B}) \cdot B = B + k - 1$ $\Rightarrow A (1 + \frac{k-1}{B}) \text{-consistent algorithm must buy before the } d\text{-th day where } d \le k$



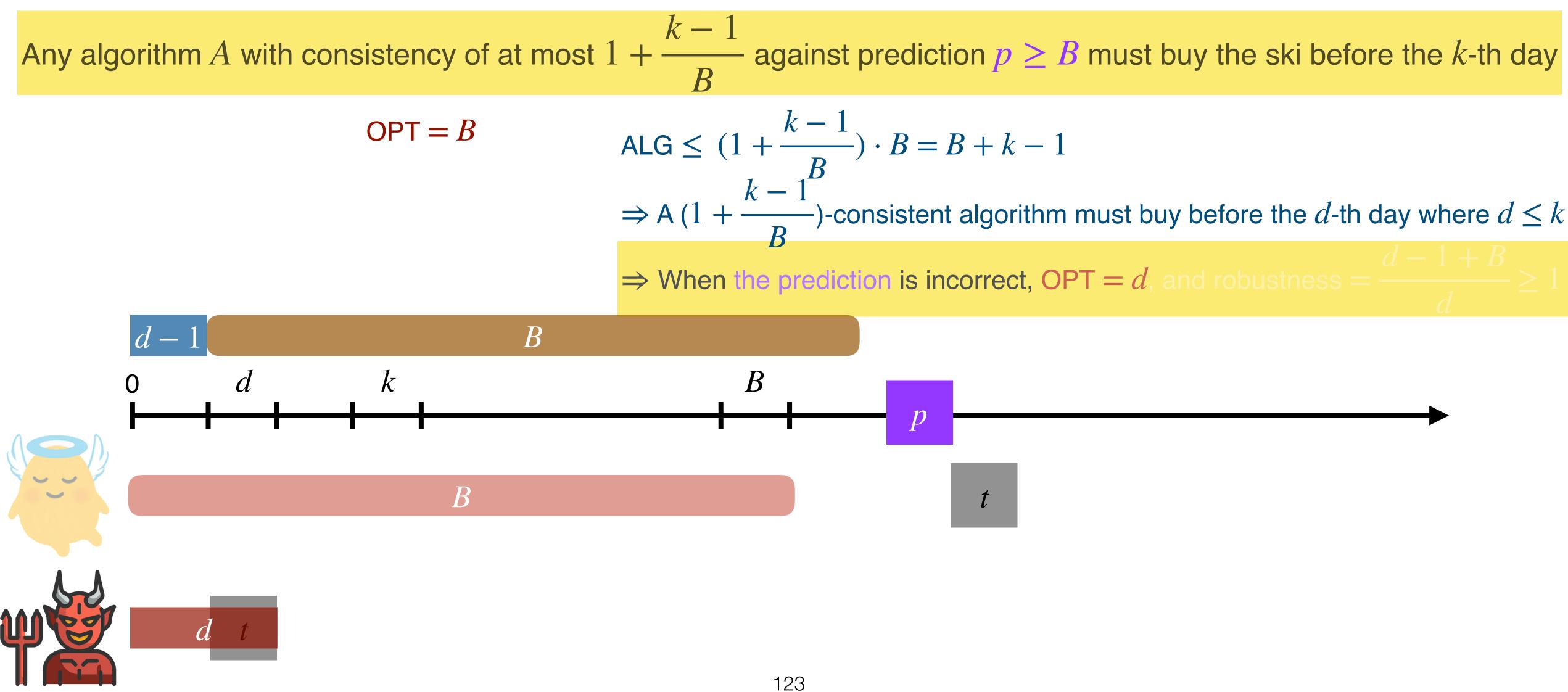




$$+\frac{k-1}{B}) \cdot B = B + k - 1$$

$$\frac{k-1}{B}$$
)-consistent algorithm must buy before the *d*-th day where the prediction is incorrect. OPT = *d*, and robustness = $\frac{d-1+d}{d}$

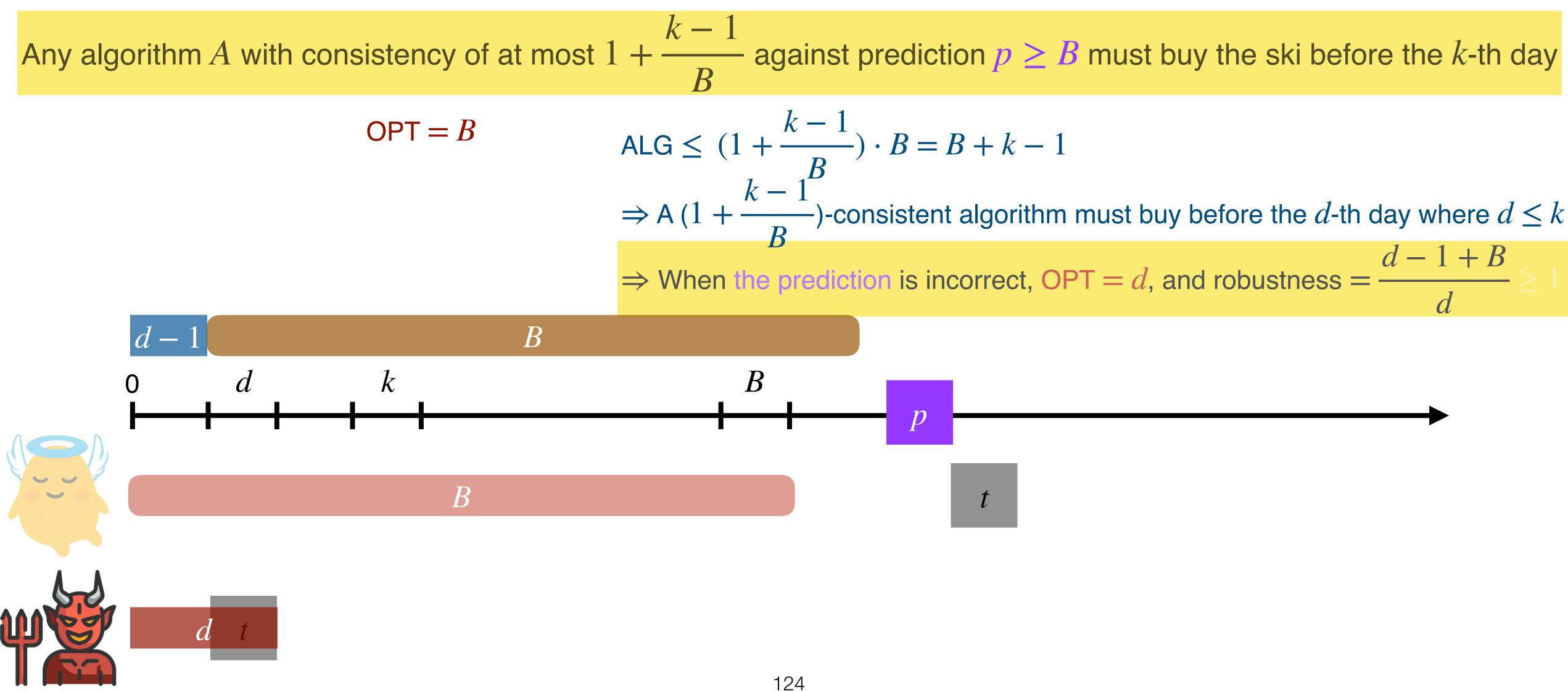




$$+\frac{k-1}{B}) \cdot B = B + k - 1$$

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)-consistent algorithm must buy before the *d*-th day where the prediction is incorrect, OPT = *d*, and robustness = $\frac{d-1+d}{d}$

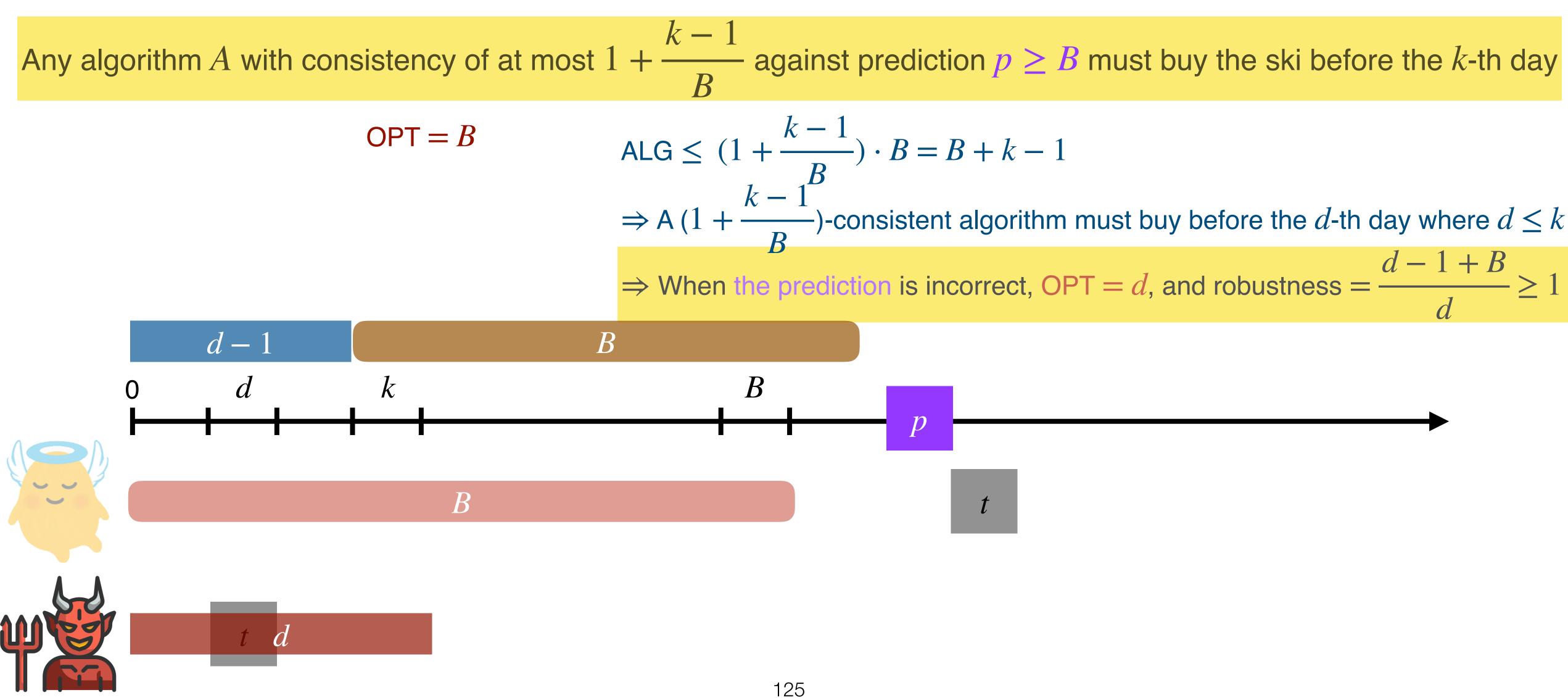




$$+\frac{k-1}{B}) \cdot B = B + k - 1$$

$$\frac{k-1}{B})$$
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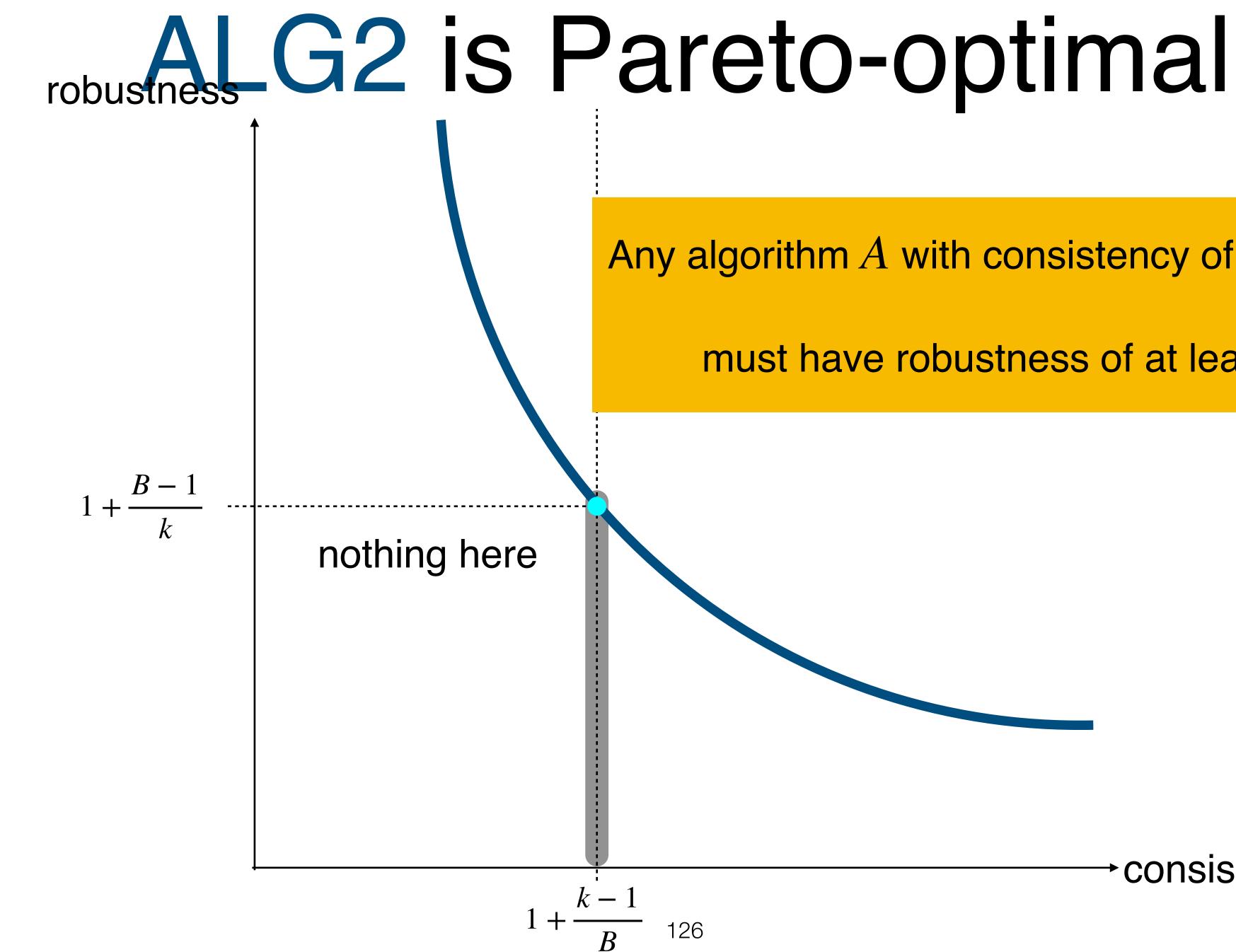




$$+\frac{k-1}{B}) \cdot B = B + k - 1$$

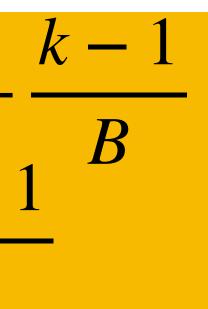
$$\frac{k-1}{B})$$
-consistent algorithm must buy before the *d*-th day where the prediction is incorrect, OPT = *d*, and robustness = $\frac{d-1+d}{d}$

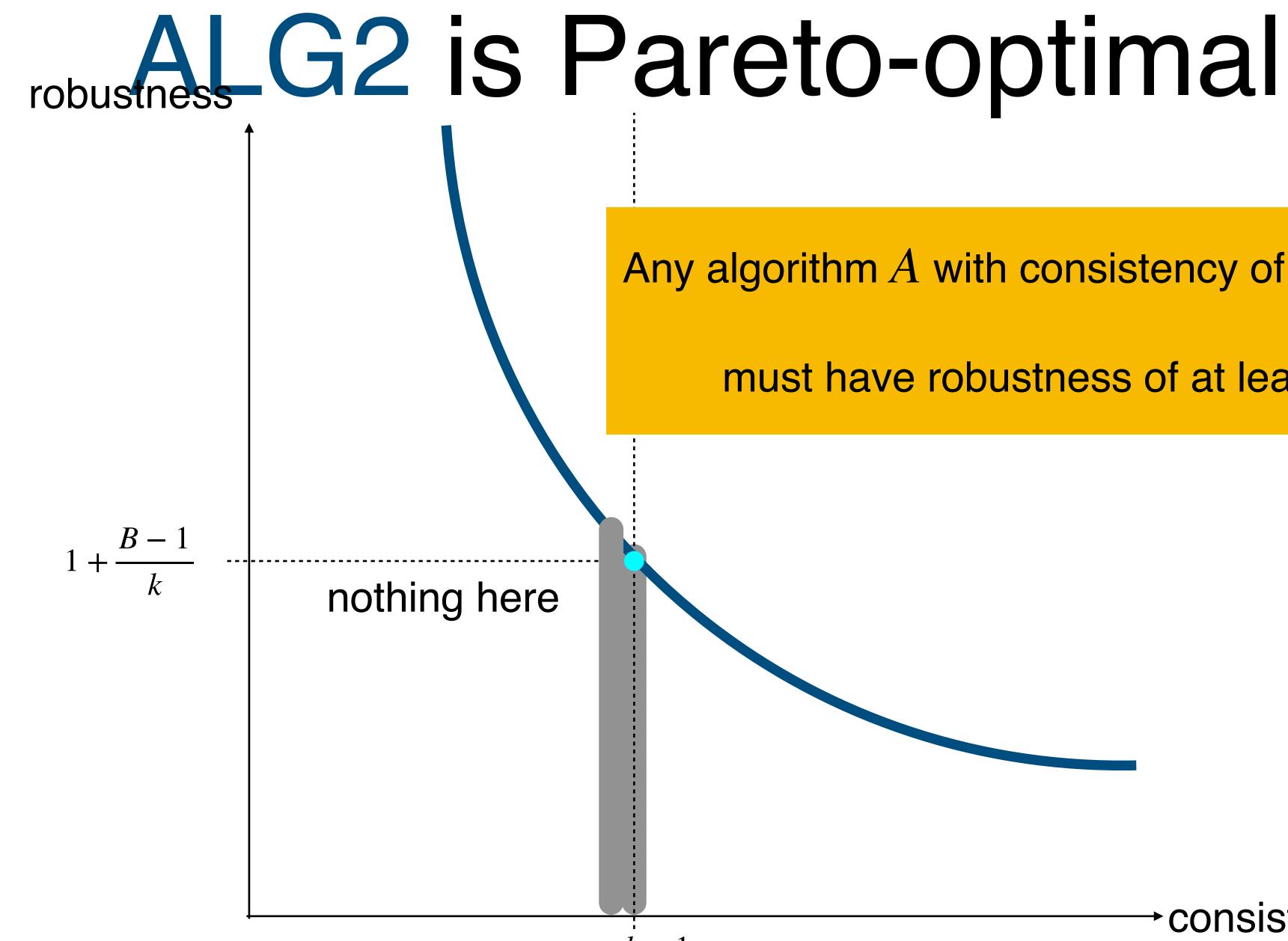




Any algorithm A with consistency of at most 1 + must have robustness of at least $1 + \frac{B-1}{-}$

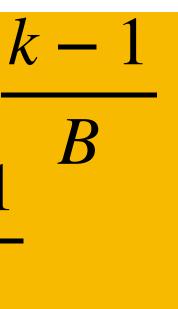
consistency

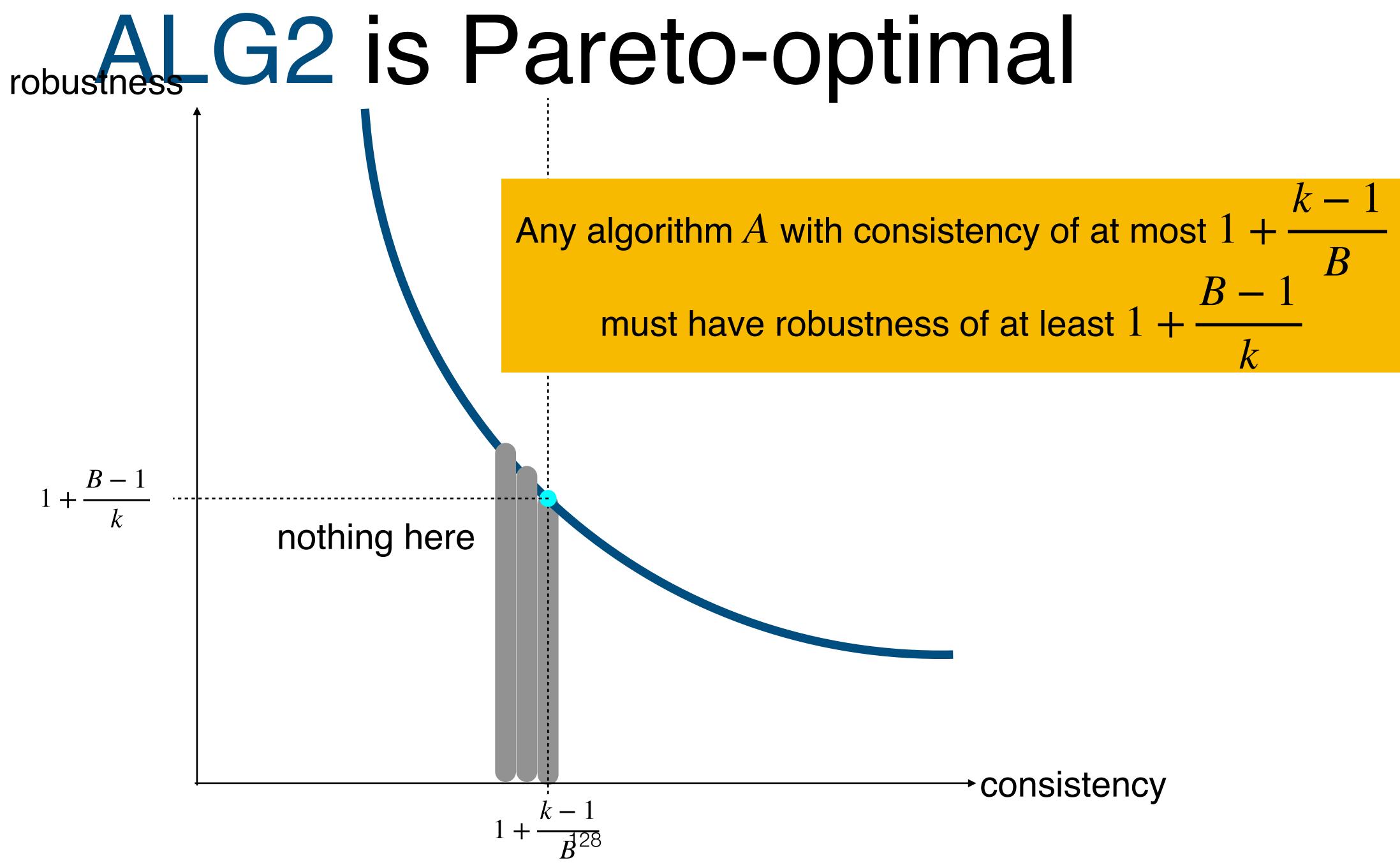




Any algorithm A with consistency of at most 1 +*B* – 1 must have robustness of at least 1 + -

 $1 + \frac{k-1}{B^{-1}27}$





robustness G2 is Pareto-optimal

nothing here

