Algorithms for Decision Support

Beyond the Worst Case

Machine-Learned Advice

Outline

- How to “trust” an advice when there is no guarantee from the advice
  - Searching
  - Ski-rental problem
    - 3 algorithms
  - Bin packing

Deal with Uncertainty

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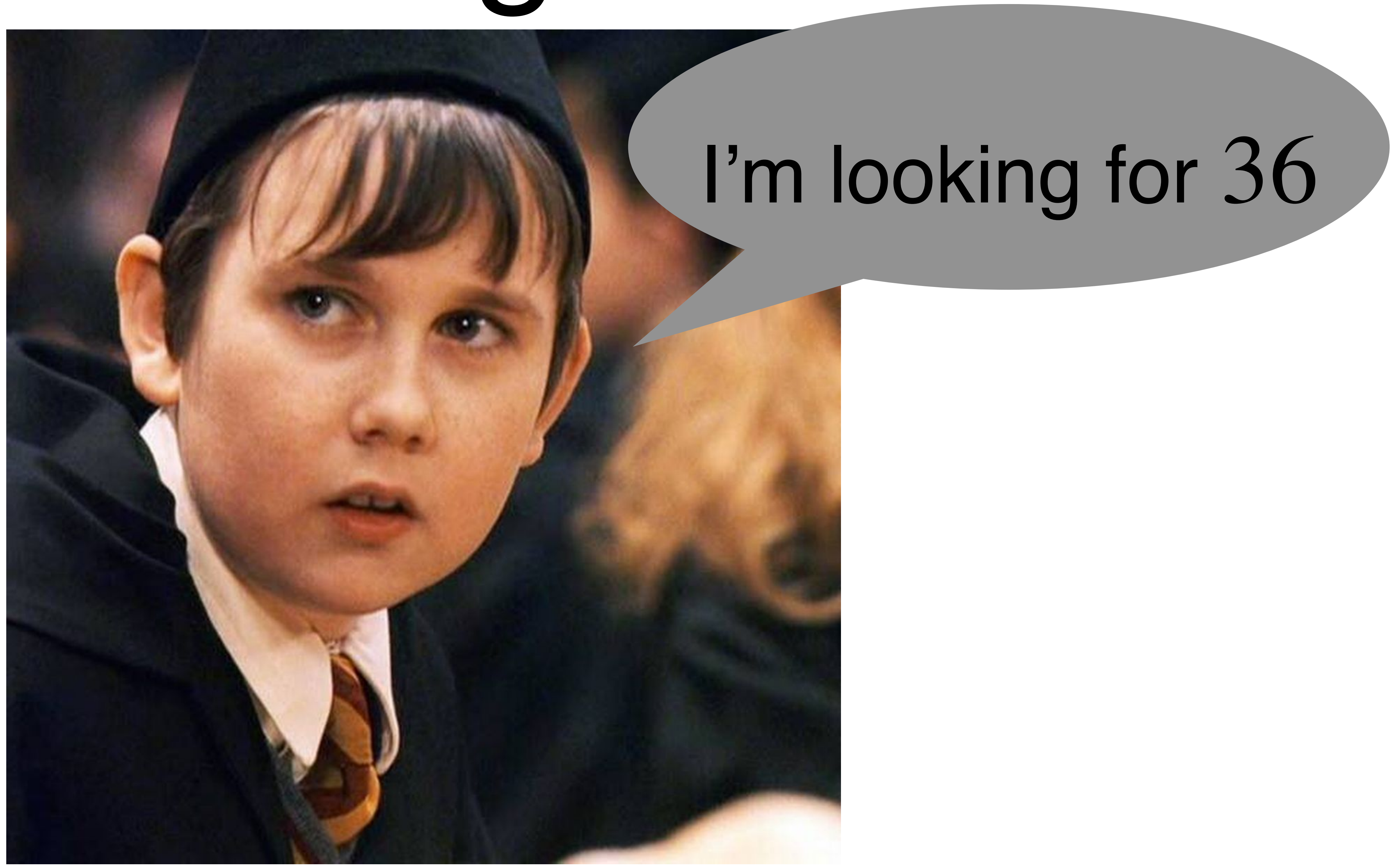
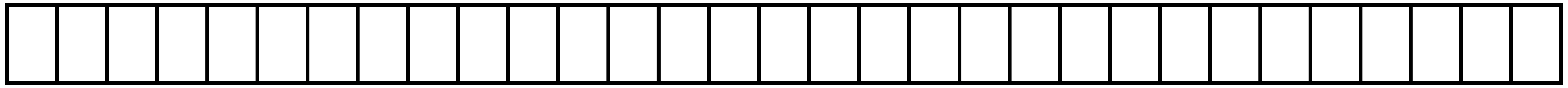
  - How if the future information can be predicted or learned by machine-learning?
    - Example: weather forecast
    - These predictions or learned information may not be 100% correct
      - Completely trust the predictions may be a disaster
      - How can we design online algorithms with this kind of (maybe) untrusted advice?

Searching with Machine-Learned Advice

- Goal: Given an ordered sequence and a requested value, find the requested value in minimum number of steps

Searching with Machine-Learned Advice



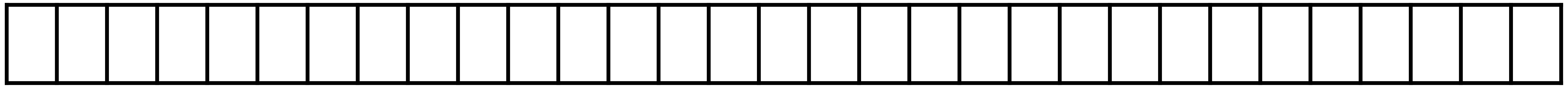


Searching with Machine-Learned Advice

I’m looking for 36

It’s at position 13!

13



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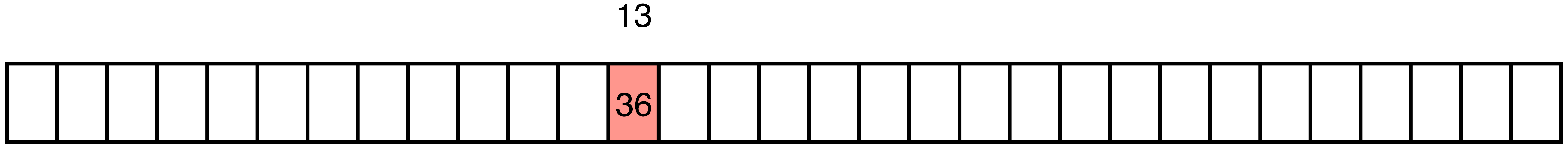

Searching with Machine-Learned Advice

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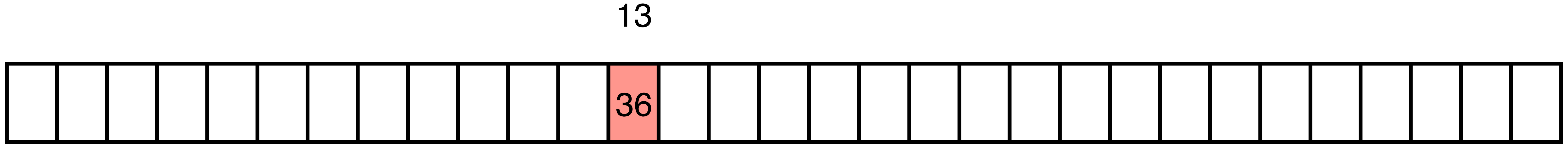
Cost = 1



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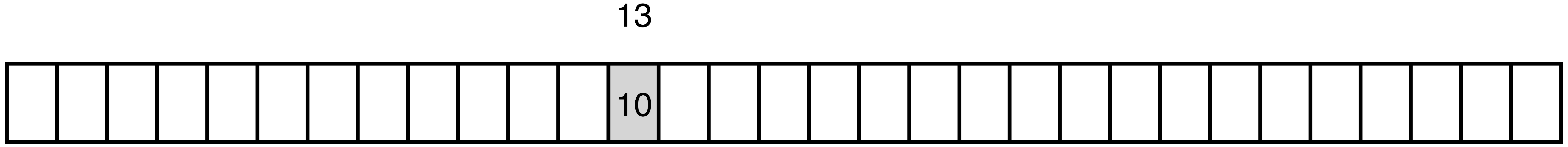

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10

Searching with Machine-Learned Advice

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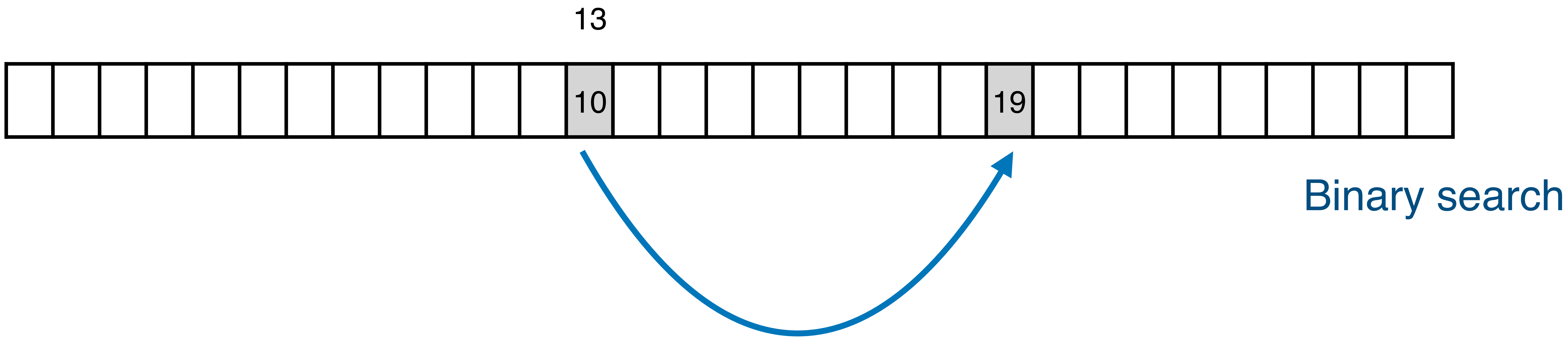

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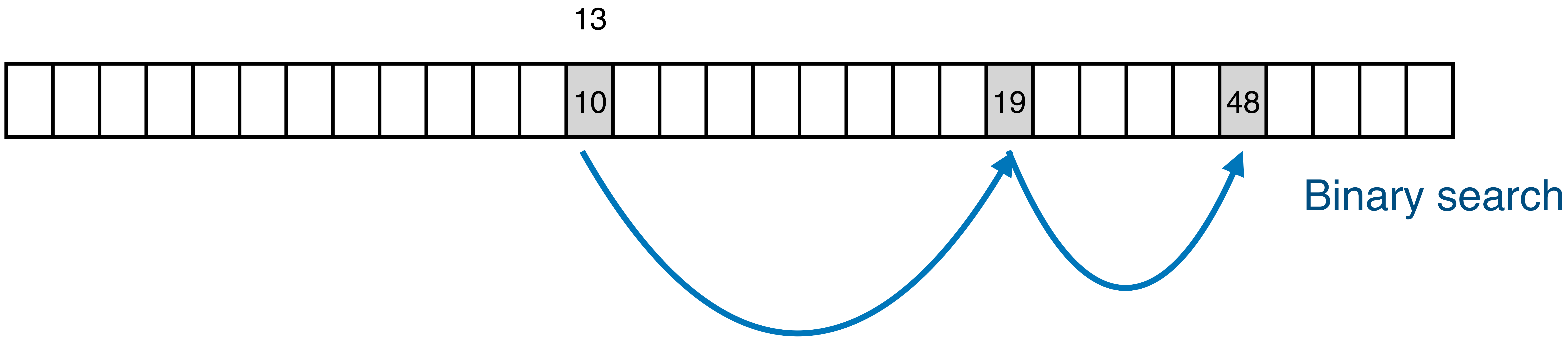

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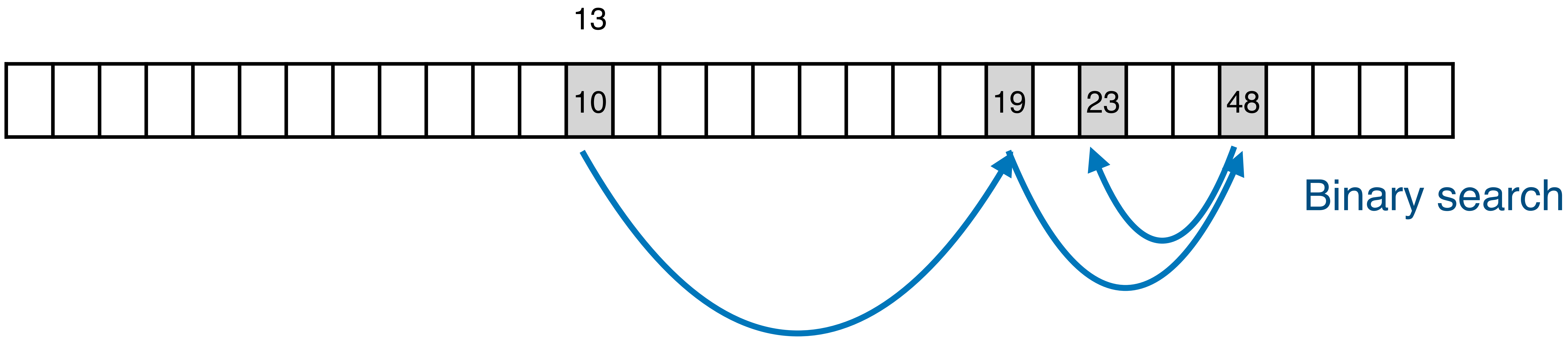

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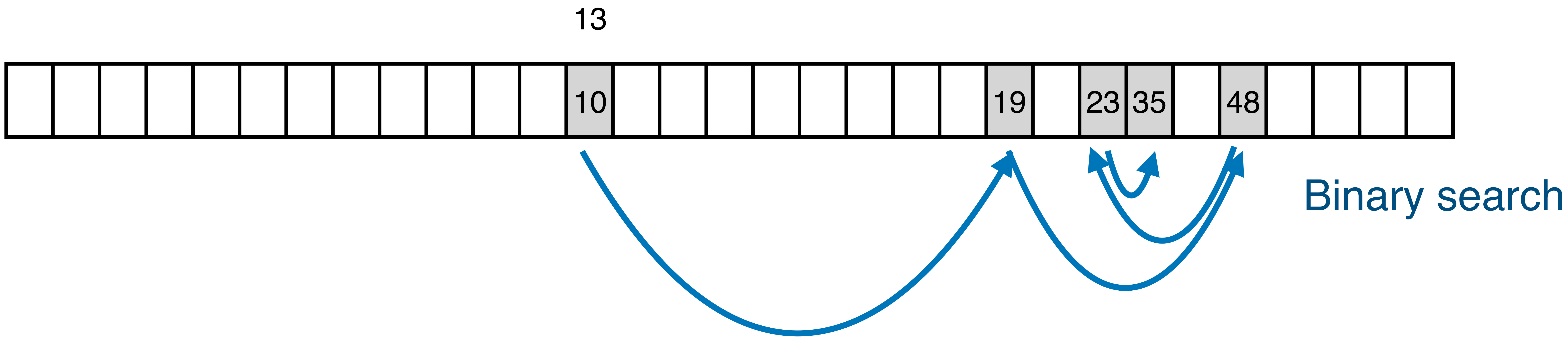

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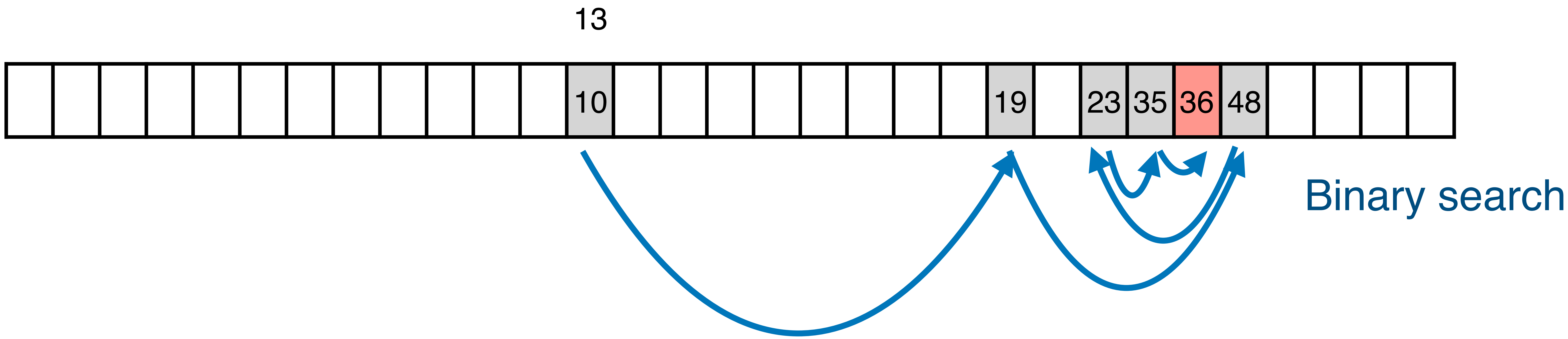

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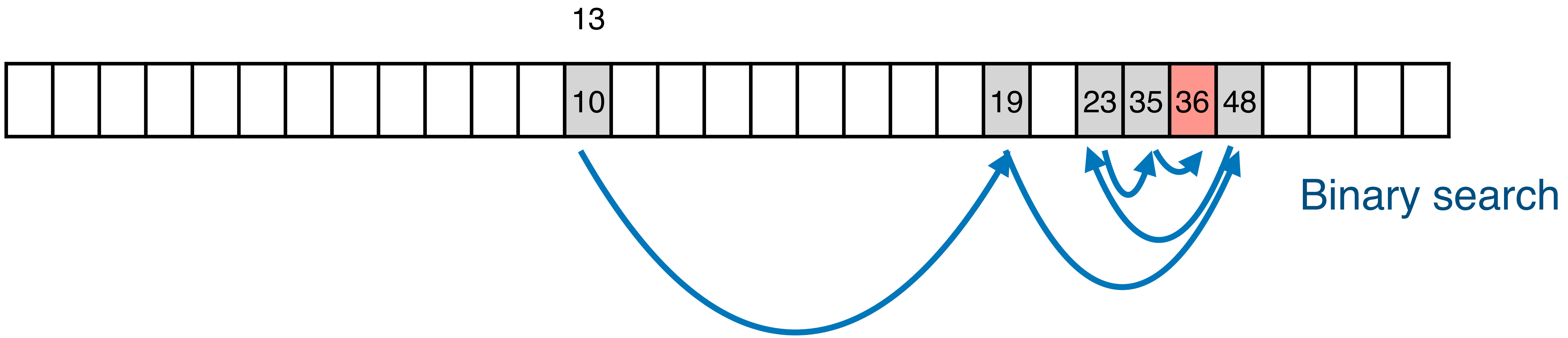
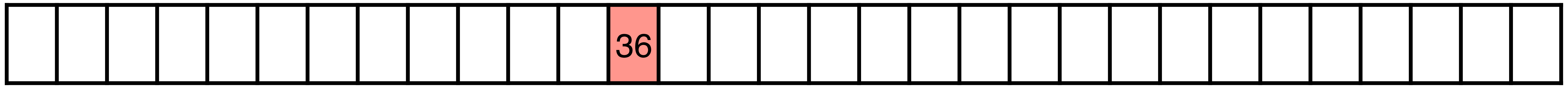
Cost = $\log n$



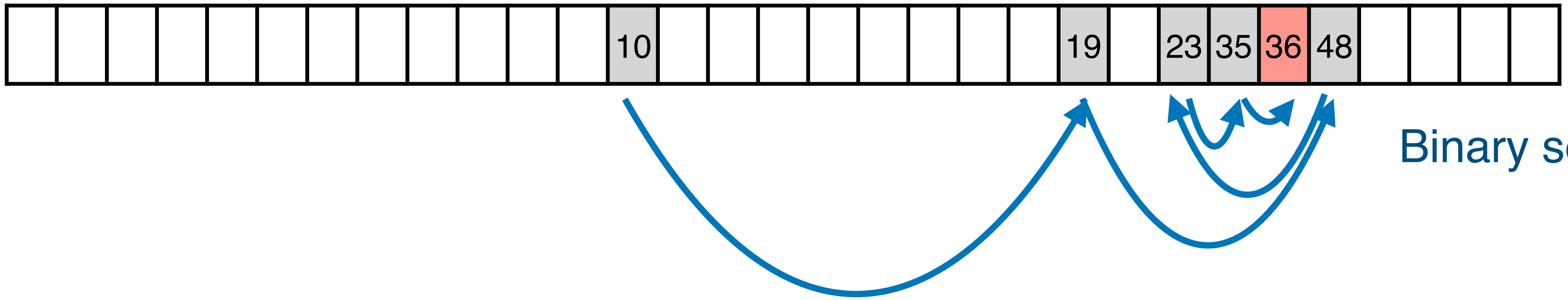
Consistency and Robustness

- We look for algorithms that
  - Are consistent: given more accurate predictions, the online algorithm should perform close to the optimal offline algorithm



Cost = 1

  - Are robust: if the prediction is wrong, the online algorithm performance should be close to the online algorithm without predictions



Cost = $\log n$

Binary search

Idea

Use predictions to improve the algorithm's performance

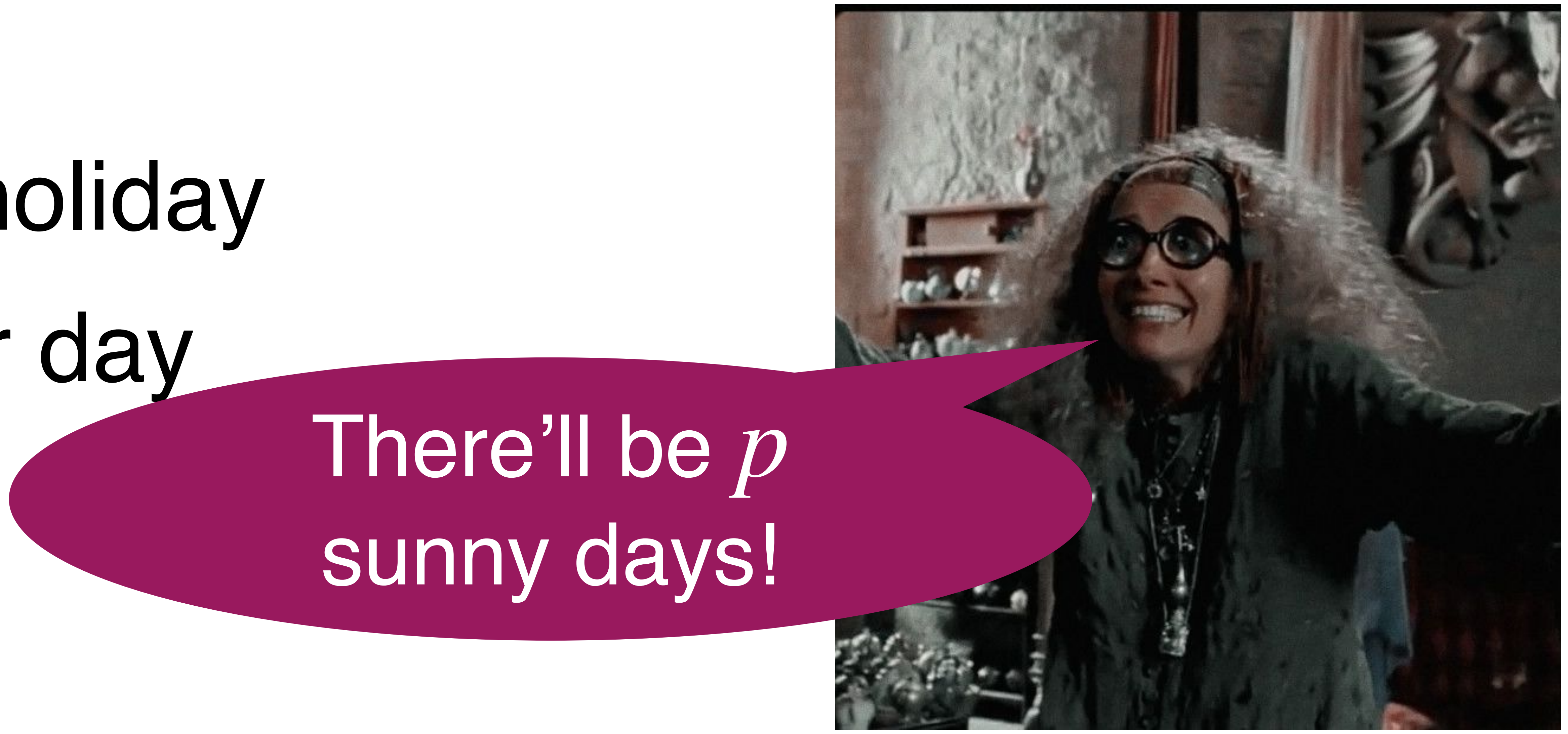
(But the prediction can be wrong)

Ski Rental

- Imagine that you are having a ski holiday
  - The price of renting a ski is 1 per day
  - The buying price for ski is B.
  - Cost: the total money you pay

- Suppose you want to spend money as little as possible. Should you buy the ski or rent it?

Ski Rental with Machine Learned Advice



- Imagine that you are having a ski holiday
  - The price of renting a ski is 1 per day
  - The buying price for ski is B.
  - Cost: the total money you pay
  - p: machine-learned prediction on the number of skiing days
- Suppose you want to spend money as little as possible. Should you buy the ski or rent it?

Ski Rental Algorithm 1

```
SKI-Rental with prediction (p)        // p: prediction on the number of skiing days
    If p ≥ B
        Buy the ski on the first day
    else ( p < B )
        Keep renting for all skiing days
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Ski Rental Algorithm 1

SKI-Rental with prediction (p) // p: prediction on the number of skiing days

```
If p ≥ B
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d: Actual number of skiing days

Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)

Ski Rental Algorithm 1

SKI-Rental with prediction (p) // p: prediction on the number of skiing days

If $p \geq B$

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else ($p < B$)

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 Good prediction		
 Bad prediction		

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 Good prediction	Advice: $p \geq B$ $\dfrac{\text{ALG}(B, d, p)}{\text{OPT}(B, d)} = \dfrac{B}{B}$	
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To be robust, we want an algorithm close to 2-competitive in this case

What Happened

- Blindly trusting the prediction can cause disasters!
  - Especially when the prediction is wrong (that is, fail to be robust)

What Happened

- Consistency: $\max_I \min_p \frac{\text{ALG}(I, p)}{\text{OPT}(I)}$ The best prediction of the input
- Robustness: $\max_I \max_p \frac{\text{ALG}(I, p)}{\text{OPT}(I)}$ The worst prediction of the input

Ski Rental Algorithm 1.5

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To be robust, we want an algorithm close to 2-competitive in this case

What Happened

- The robustness is still bad when the prediction is wrong
  - Especially when the algorithm is tricked into buying the ski

Trustness Parameter

- We introduce a parameter k to indicate how much the algorithm trust the advice
  - $k \in [1, B]$
    - $k = 1$: the algorithm fully trusts the advice
    - $k = B$: the algorithm does not trust the advice at all

Ski Rental Algorithm 2

SKI-Rental with prediction (p, k)

If $p \geq B$

Keep renting until the k-th day // k is our "trust parameter"

else ($p < B$)

Keep renting until the B-th day

Ski Rental Algorithm 2

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Fully trust the prediction

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Does not trust the prediction at all

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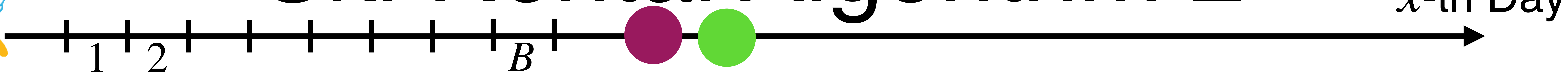

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	Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)
 Good prediction	Advice: $p \geq B$ $\frac{(k-1)+B}{B}$	Advice: $p < B$
 Bad prediction	Advice: $p < B$	Advice: $p \geq B$

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 Bad prediction	Advice: $p < B$	Advice: $p \geq B$

Ski Rental Algorithm 2

SKI-Rental with prediction (p, k)

If $p \geq B$

Keep renting until the k-th day // k is our "trust parameter"

else ($p < B$)

Keep renting until the B-th day

d: Actual number of skiing days

	Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)
Good prediction	Advice: $p \geq B$ $\frac{(k-1)+B}{B}$	Advice: $p < B$
Bad prediction	Advice: $p < B$	Advice: $p \geq B$

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	Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)
 Good prediction	Advice: $p \geq B$ $\frac{(k-1)+B}{B}$	Advice: $p < B$ $\frac{d}{d}$
 Bad prediction	Advice: $p < B$	Advice: $p \geq B$

Ski Rental Algorithm 2

SKI-Rental with prediction (p, k)
  If $p \geq B$
    Keep renting until the k-th day   // k is our "trust parameter"
  else ($p < B$)
    Keep renting until the B-th day

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	Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)
 Good prediction	Advice: $p \geq B$ $\frac{(k-1)+B}{B}$	Advice: $p < B$ $\frac{d}{d}$
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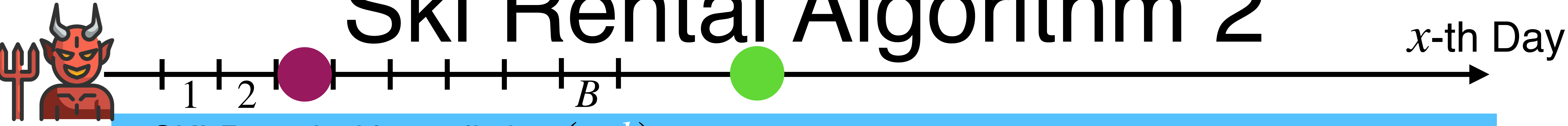
Ski Rental Algorithm 2



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	Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)
Good prediction	Advice: $p \geq B$ $\frac{(k-1)+B}{B}$	Advice: $p < B$ $\frac{d}{d}$
Bad prediction	Advice: $p < B$ $\frac{(B-1)+B}{B} = 2 - \frac{1}{B}$	Advice: $p \geq B$

Ski Rental Algorithm 2

SKI-Rental with prediction (p, k)
    If $p \geq B$
        Keep renting until the k-th day   // k is our "trust parameter"
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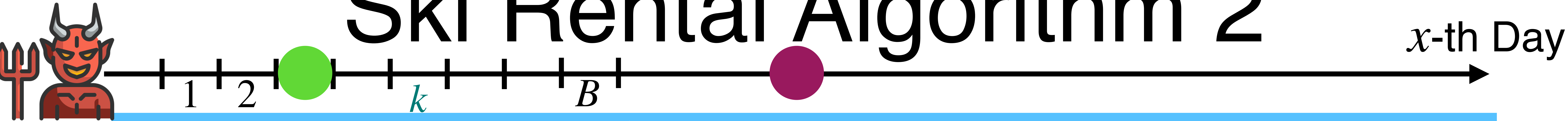
Ski Rental Algorithm 2



SKI-Rental with prediction (p, k)

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Ski Rental Algorithm 2

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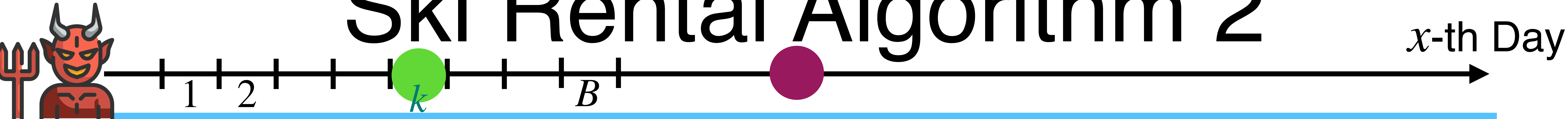
 Bad prediction	Advice: $p < B$ $\frac{(B-1)+B}{B} = 2 - \frac{1}{B}$	Advice: $p \geq B$ $\frac{d}{d}$

Ski Rental Algorithm 2



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Ski Rental Algorithm 2

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Good prediction	Advice: $p \geq B$ $\frac{(k-1)+B}{B}$	Advice: $p < B$ $\frac{d}{d}$
Bad prediction	Advice: $p < B$ $\frac{(B-1)+B}{B} = 2 - \frac{1}{B}$	Advice: $p \geq B$ $\frac{(k-1)+B}{k} = 1 + \frac{B-1}{k}$

Ski Rental Algorithm 2

SKI-Rental with prediction (p, k)
If $p \geq B$
Keep renting until the k-th day // k is our "trust parameter"
else ($p < B$)
Keep renting until the B-th day

d: Actual number of skiing days

	Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)
 Good prediction	Advice: $p \geq B$ $\frac{(k-1)+B}{B} = 1 + \frac{k-1}{B}$	Advice: $p < B$ $\frac{d}{d} = 1$
 Bad prediction	Advice: $p < B$ $\frac{(B-1)+B}{B} = 2 - \frac{1}{B}$	Advice: $p \geq B$ $\frac{(k-1)+B}{k} = 1 + \frac{B-1}{k}$

Ski Rental Algorithm 2

SKI-Rental with prediction (p, k)
- If $p \geq B$
  - Keep renting until the k-th day // k is our "trust parameter"
- else ($p < B$)
  - Keep renting until the B-th day

d: Actual number of skiing days

	Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)
 Good prediction **Consistency**	Advice: $p \geq B$ $\frac{(k-1)+B}{B} = 1 + \frac{k-1}{B}$	Advice: $p < B$ $\frac{d}{d} = 1$
 Bad prediction **Robustness**	Advice: $p < B$ $\frac{(B-1)+B}{B} = 2 - \frac{1}{B}$	Advice: $p \geq B$ $\frac{(k-1)+B}{k} = 1 + \frac{B-1}{k}$

What Happened

- Use a trust parameter to partially trust the prediction
  - In this case, trust parameter $k \in [1,B]$
    - The smaller k is, the more the algorithm trusts the prediction
- ALG2 is $(1 + \frac{k-1}{B})$-consistent and $(1 + \frac{B-1}{k})$-robust
  - When $k = 1$, the consistency is 1 and the robustness is B
  - When $k = B$, the consistency and the robustness are both $2 - \frac{1}{B}$

Prediction Error η

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- Absolute error $\eta_1 = |p - d|$

Prediction Error η

- Absolute error $\eta_1 = |p - d|$
- Squared error $\eta_2 = |p - d|^2$

Prediction Error η

- Absolute error $\eta_1 = |p - d|$
- Squared error $\eta_2 = |p - d|^2$
- Classification error $\eta_c = 1$ if $p \neq d$
- ……

Ski Rental Algorithms Revisit: $\eta_1 = | p - d |$

- $\eta_1 = | p - d |$

Algorithm 3

If $p \geq B$	then **Buy** on the $\lceil B \cdot (1-\alpha) + \varepsilon \rceil$-th day
Else	**Buy** on the $\lceil \frac{B}{1-\alpha} + \varepsilon \rceil$-th day

Algorithm 3

If $p \geq B$ then **Buy** on the $\lceil B \cdot (1-\alpha) + \varepsilon \rceil$-th day

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Algorithm 3

If $p \geq B$ then **Buy** on the $\lceil B\cdot(1-\alpha)+\varepsilon \rceil$-th day

Else **Buy** on the $\lceil \frac{B}{1-\alpha}+\varepsilon \rceil$-th day

Big α

Algorithm 3

`If` $p \geq B$ `then` **`Buy`** `on the` $\lceil B \cdot (1-\alpha)+\varepsilon \rceil$`-th day`

`Else` **`Buy`** `on the` $\lceil \frac{B}{1-\alpha} + \varepsilon \rceil$`-th day`

Small α

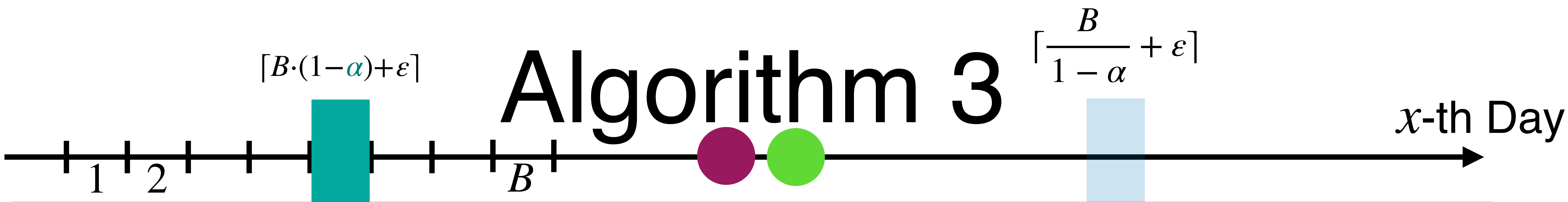
Algorithm 3

`If` $p \geq B$ `then` **`Buy`** `on the` $\lceil B\cdot(1-\alpha)+\varepsilon\rceil$`-th day`

`Else` **`Buy`** `on the` $\lceil \frac{B}{1-\alpha} + \varepsilon \rceil$`-th day`

	Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)
 Good prediction		
 Bad prediction		

Algorithm 3



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Else **Buy** on the $\lceil \frac{B}{1-\alpha}+\varepsilon \rceil$-th day

	Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)
 Good prediction	Advice: $p \geq B$ $\frac{B(1-\alpha)+B}{B} = 2-\alpha$	
 Bad prediction		

Algorithm 3

If $p \geq B$ then **Buy** on the $\lceil B\cdot(1-\alpha)+\varepsilon \rceil$-th day

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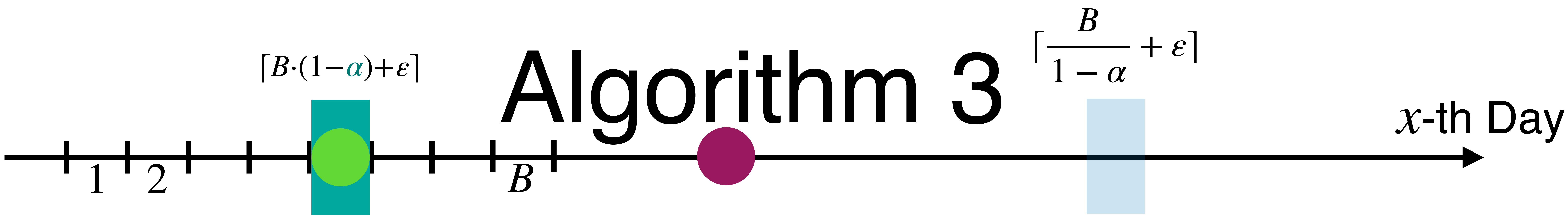
Algorithm 3



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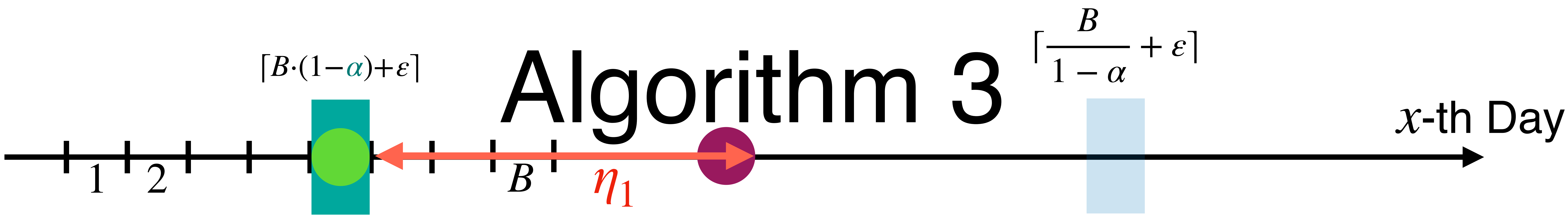
	Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)
Bad prediction		Advice: $p \geq B$

Algorithm 3



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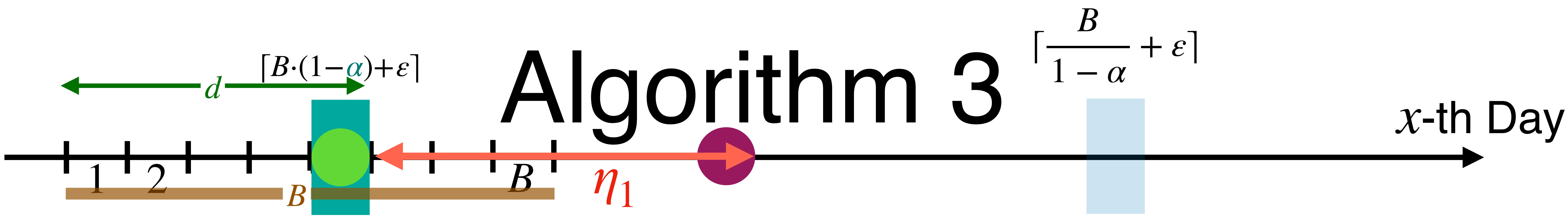
	Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)
Bad prediction		Advice: $p \geq B \Rightarrow \eta_1 = p - d$

Algorithm 3



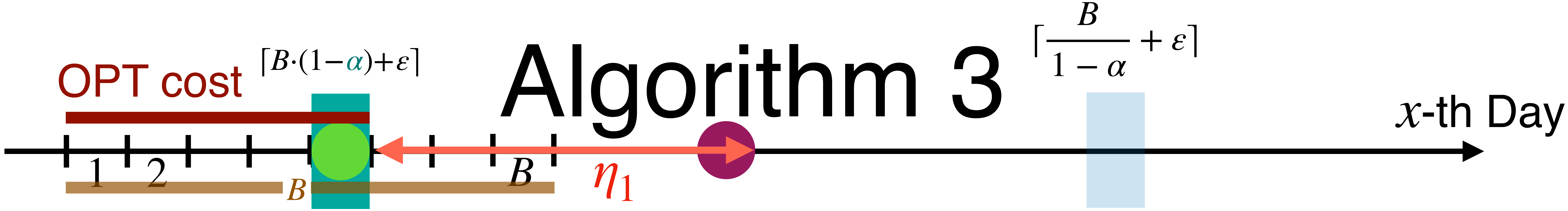

```
If p ≥ B      then Buy on the ⌈B·(1−α)+ε⌉-th day
Else               Buy on the ⌈B/(1−α)+ε⌉-th day
```

If $p \geq B$ then **Buy** on the $\lceil B\cdot(1-\alpha)+\varepsilon\rceil$-th day

Else **Buy** on the $\lceil \frac{B}{1-\alpha}+\varepsilon\rceil$-th day

	Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)
Bad prediction		Advice: $p \geq B \Rightarrow \eta_1 = p - d$ $\Rightarrow B \leq d + \eta_1$

Algorithm 3



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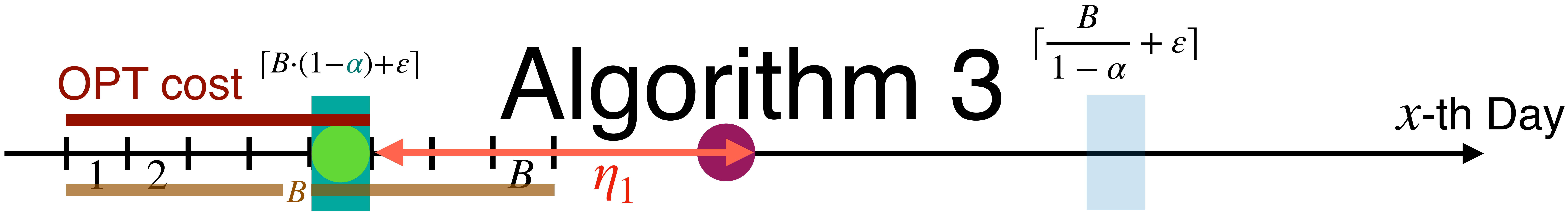
Algorithm 3



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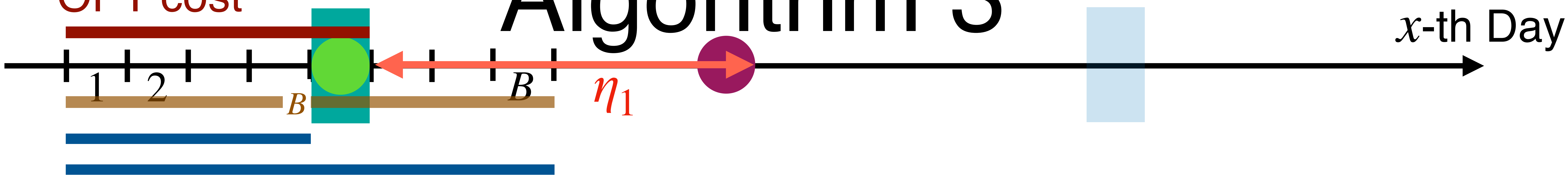
Algorithm 3



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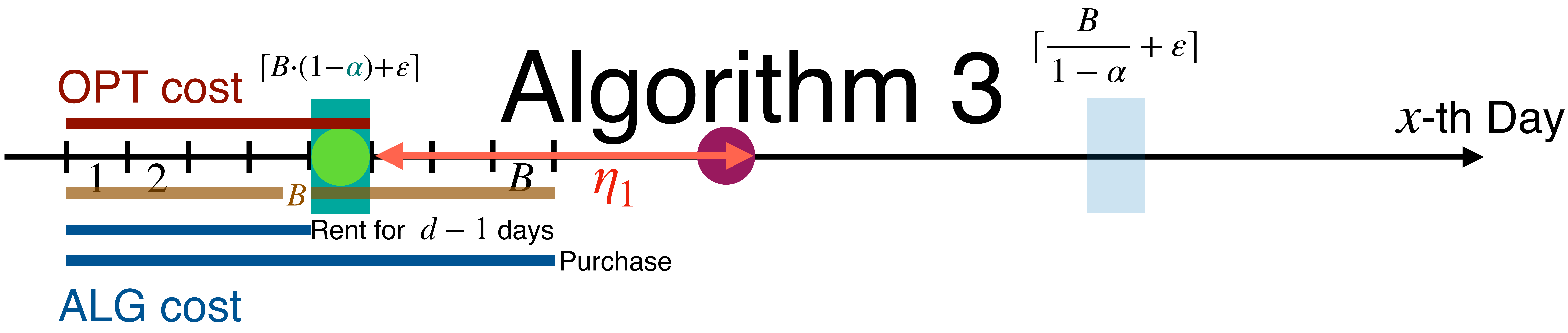
Algorithm 3



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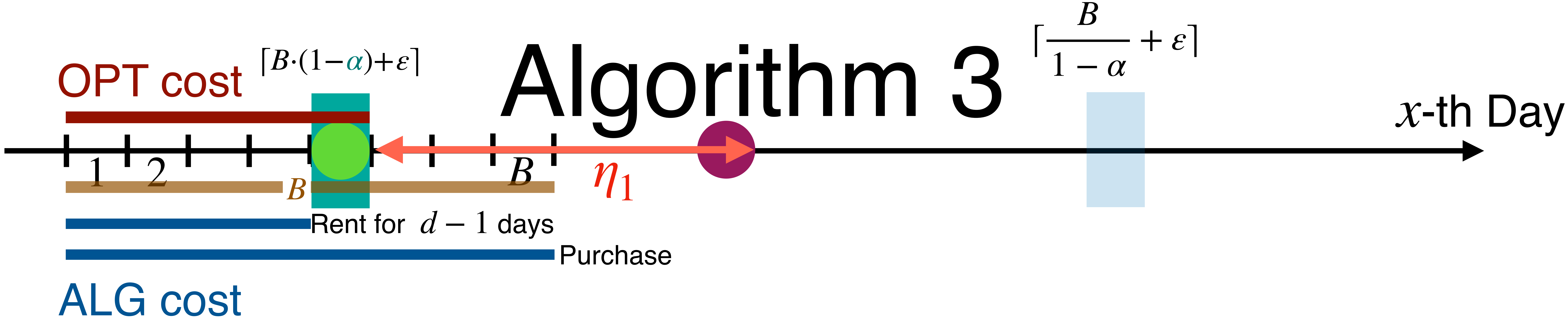
Algorithm 3



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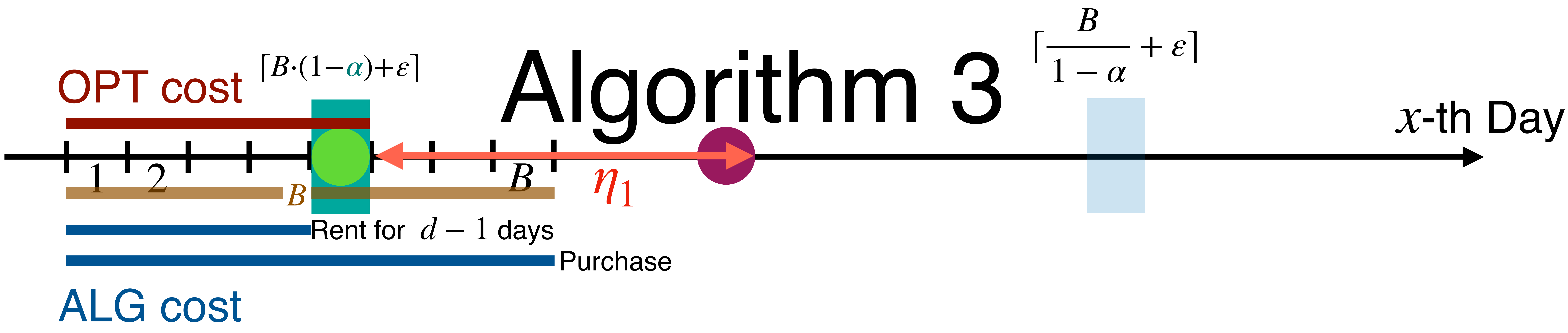
	Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)
 Bad prediction		Advice: $p \geq B \Rightarrow \eta_1 = p - d$ $\Rightarrow B \leq d + \eta_1 = \text{OPT} + \eta_1$ ALG buys ALG $\leq B(1-\alpha) + B$ $\leq (2-\alpha)(\text{OPT} + \eta_1)$

Algorithm 3



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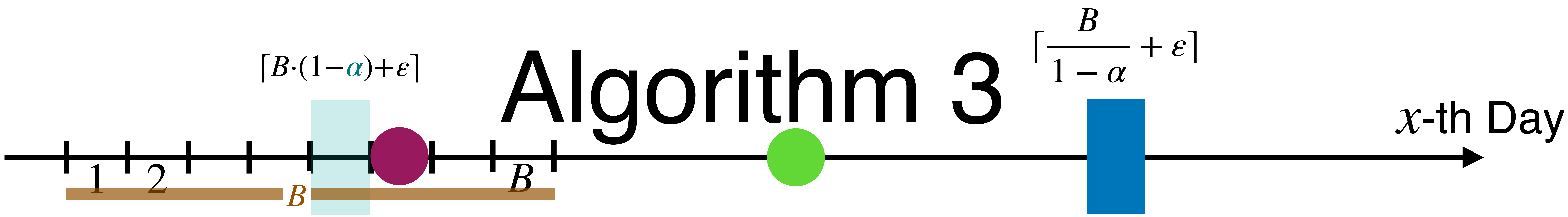
Algorithm 3



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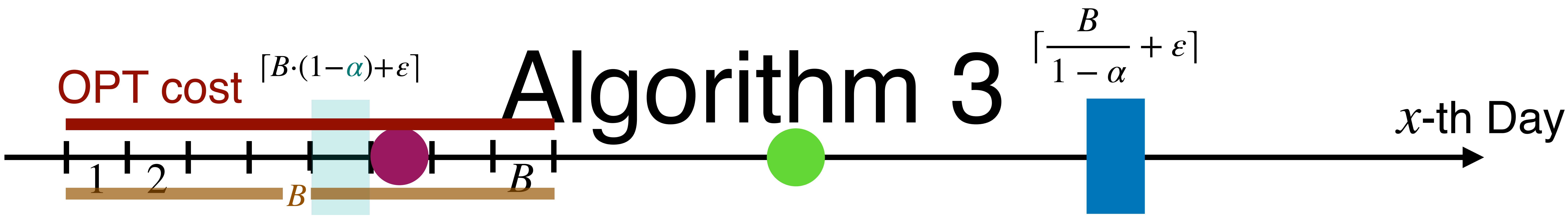
Algorithm 3



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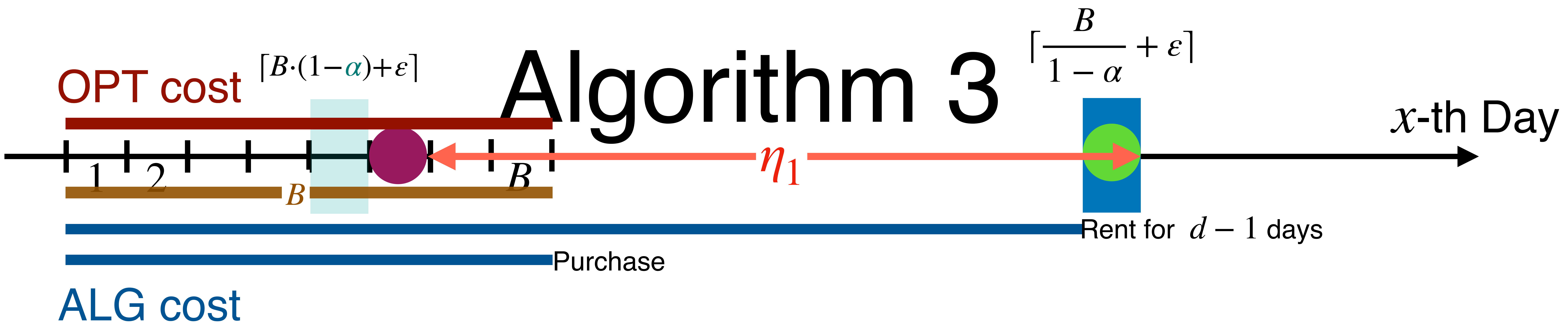
Algorithm 3



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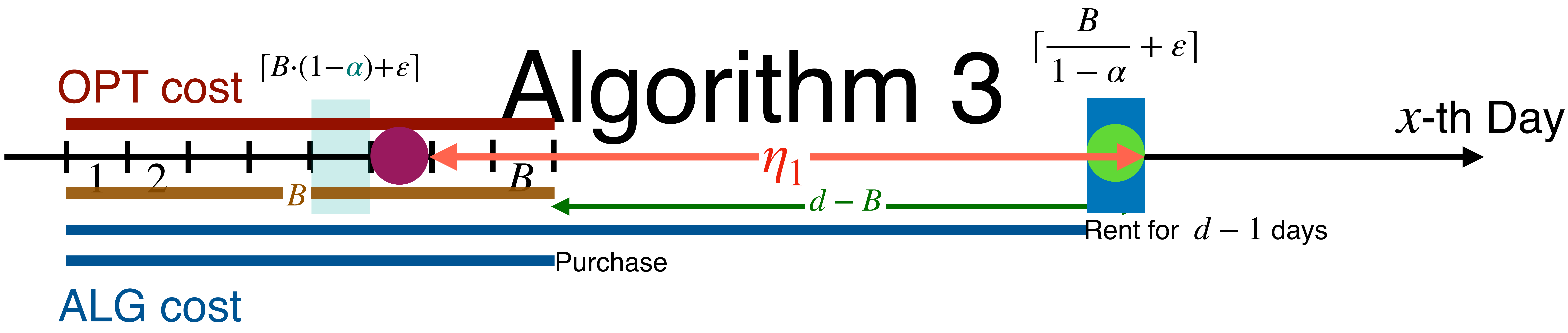
Algorithm 3



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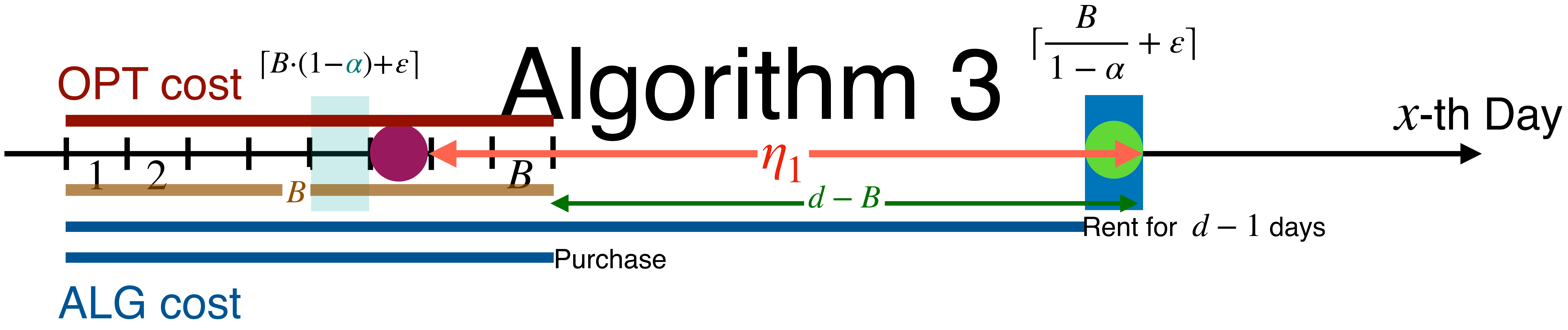

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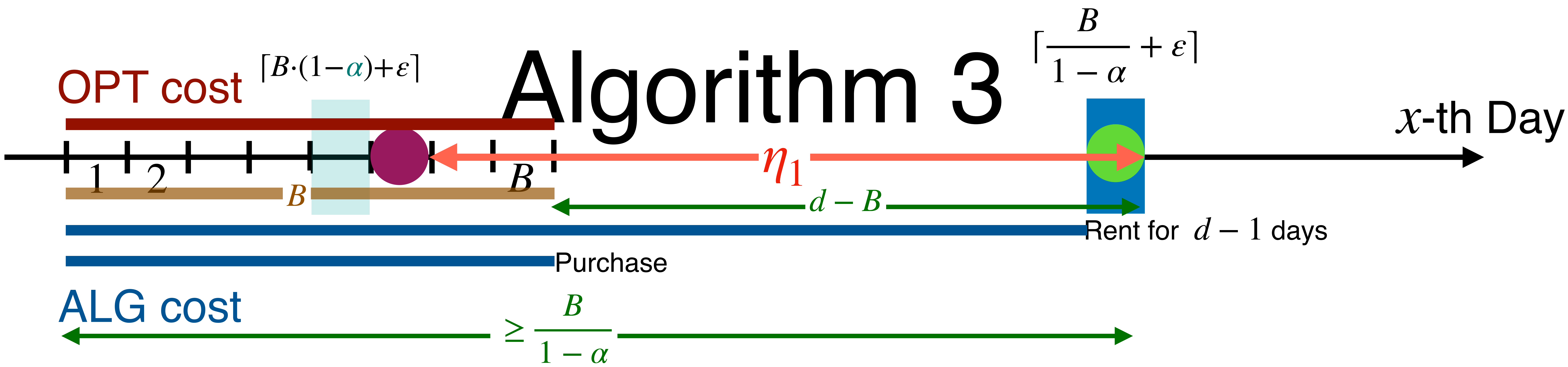
Algorithm 3



If $p \geq B$ then **Buy** on the $\lceil B\cdot(1-\alpha)+\varepsilon \rceil$-th day

Else **Buy** on the $\lceil \frac{B}{1-\alpha} + \varepsilon \rceil$-th day

	Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)
 Bad prediction	Advice: $p < B \Rightarrow \eta_1 = d - p \geq d - B$ If $d \geq \frac{B}{1-\alpha}$, ALG buys $(\eta_1 \geq d - B \geq \frac{B}{1-\alpha} - B = \frac{\alpha \cdot B}{1-\alpha})$ $\Rightarrow$ ALG $\leq B + \frac{B}{1-\alpha} \leq \text{OPT} + \frac{\eta_1}{\alpha}$	Advice: $p \geq B \Rightarrow \eta_1 = p - d$ $\Rightarrow B \leq d + \eta_1 = \text{OPT} + \eta_1$ ALG buys ALG $\leq B(1-\alpha) + B$ $\leq (2-\alpha)(\text{OPT} + \eta_1)$

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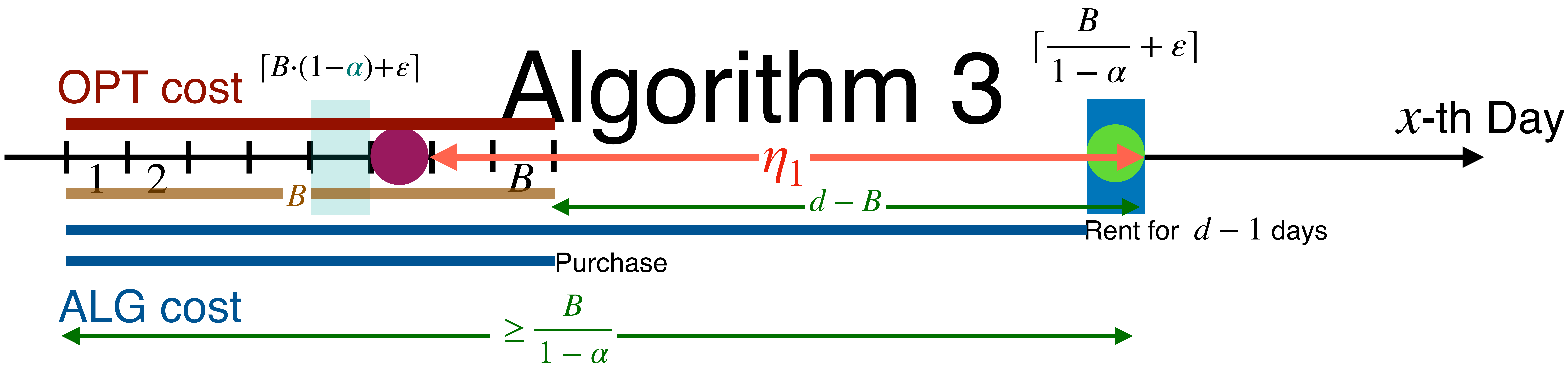

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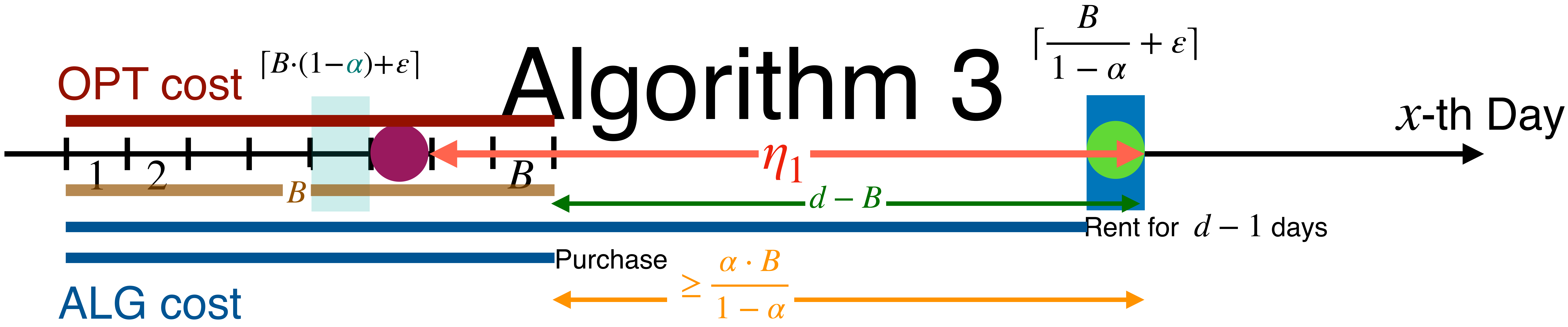

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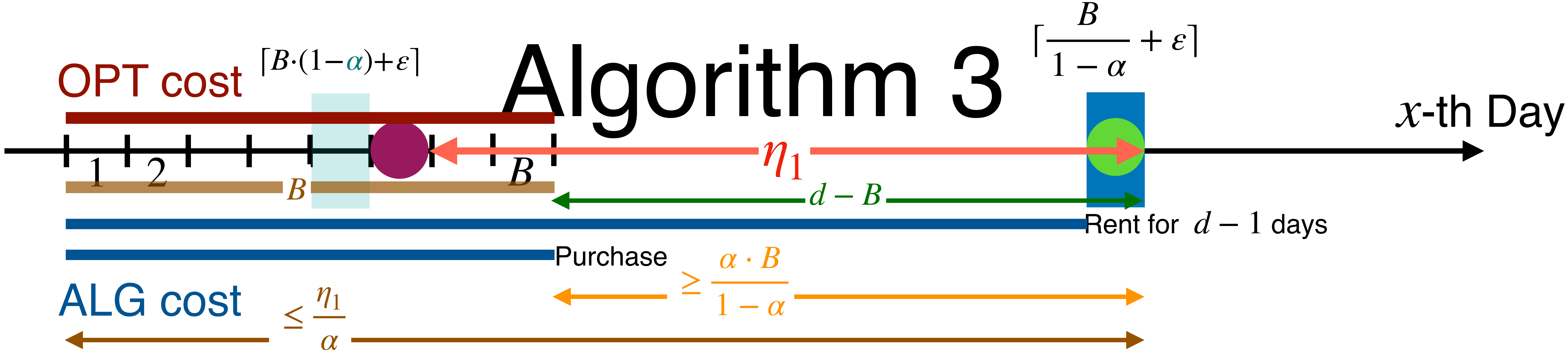

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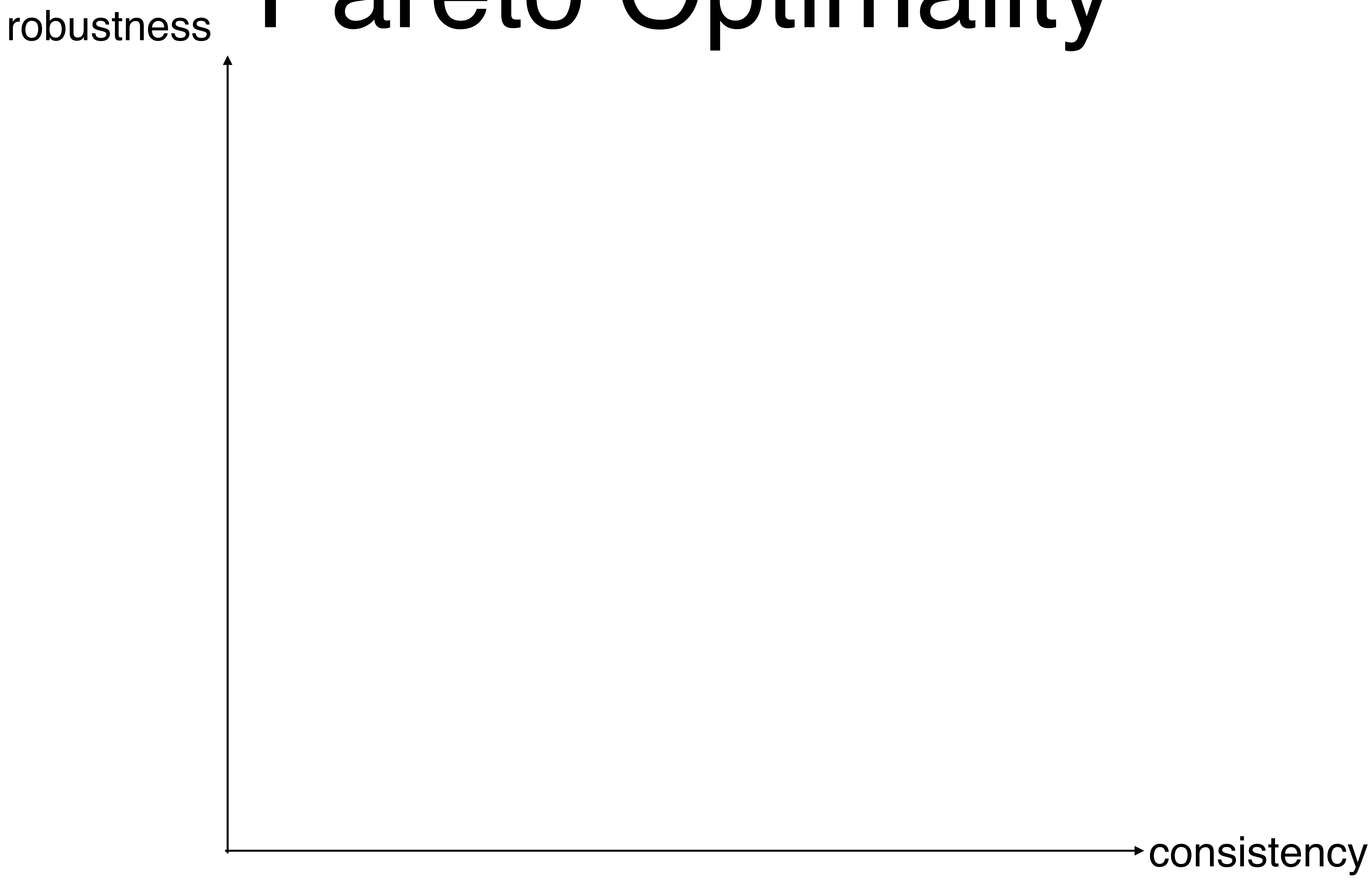
What Happened

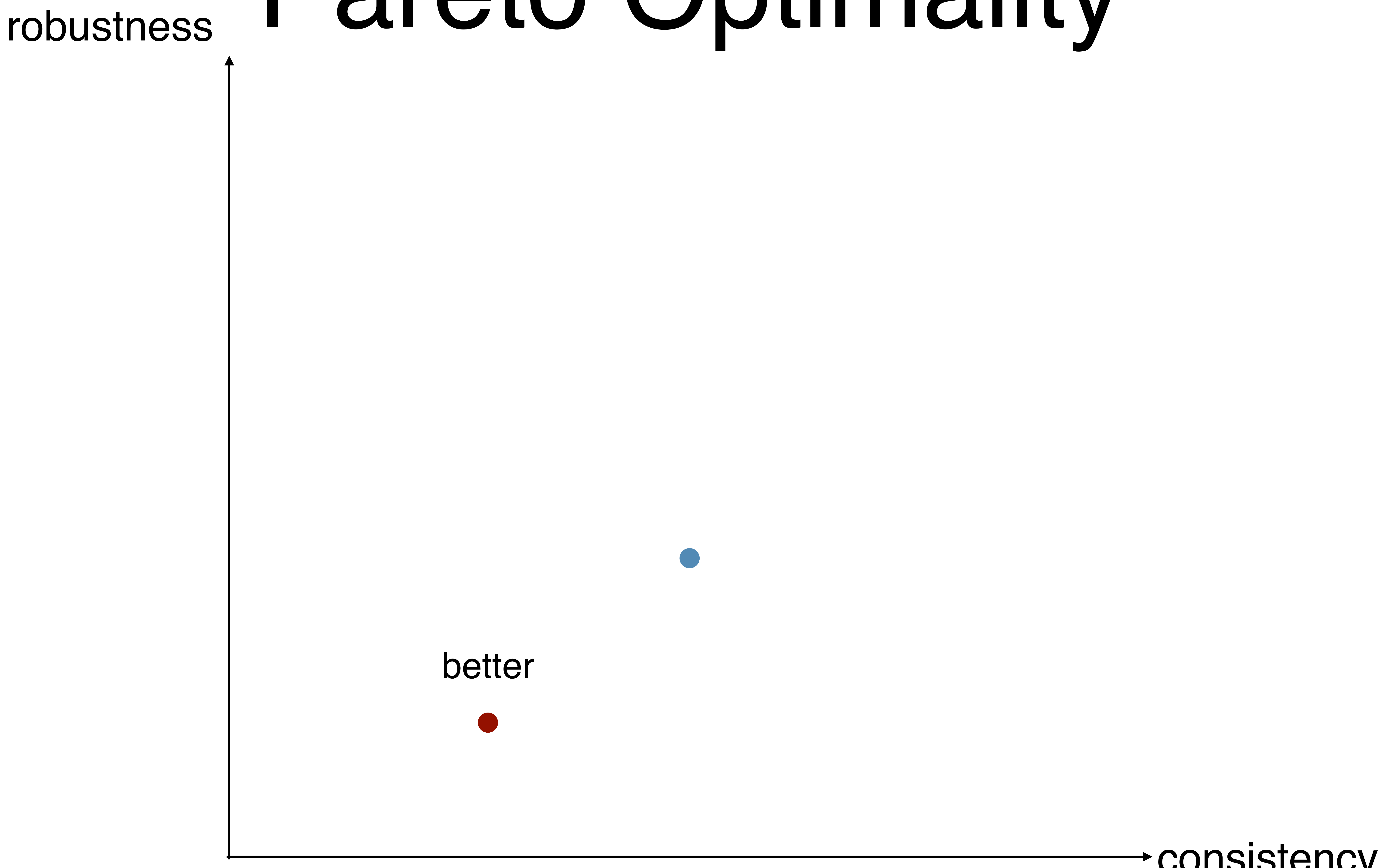
- We can use the absolute error measure of the prediction to represent the robustness
  - More specifically, ALG3's cost is a function of 1) the error measure, 2) the trust parameter, and 3) OPT cost

Pareto Optimality



Pareto Optimality

Pareto Optimality



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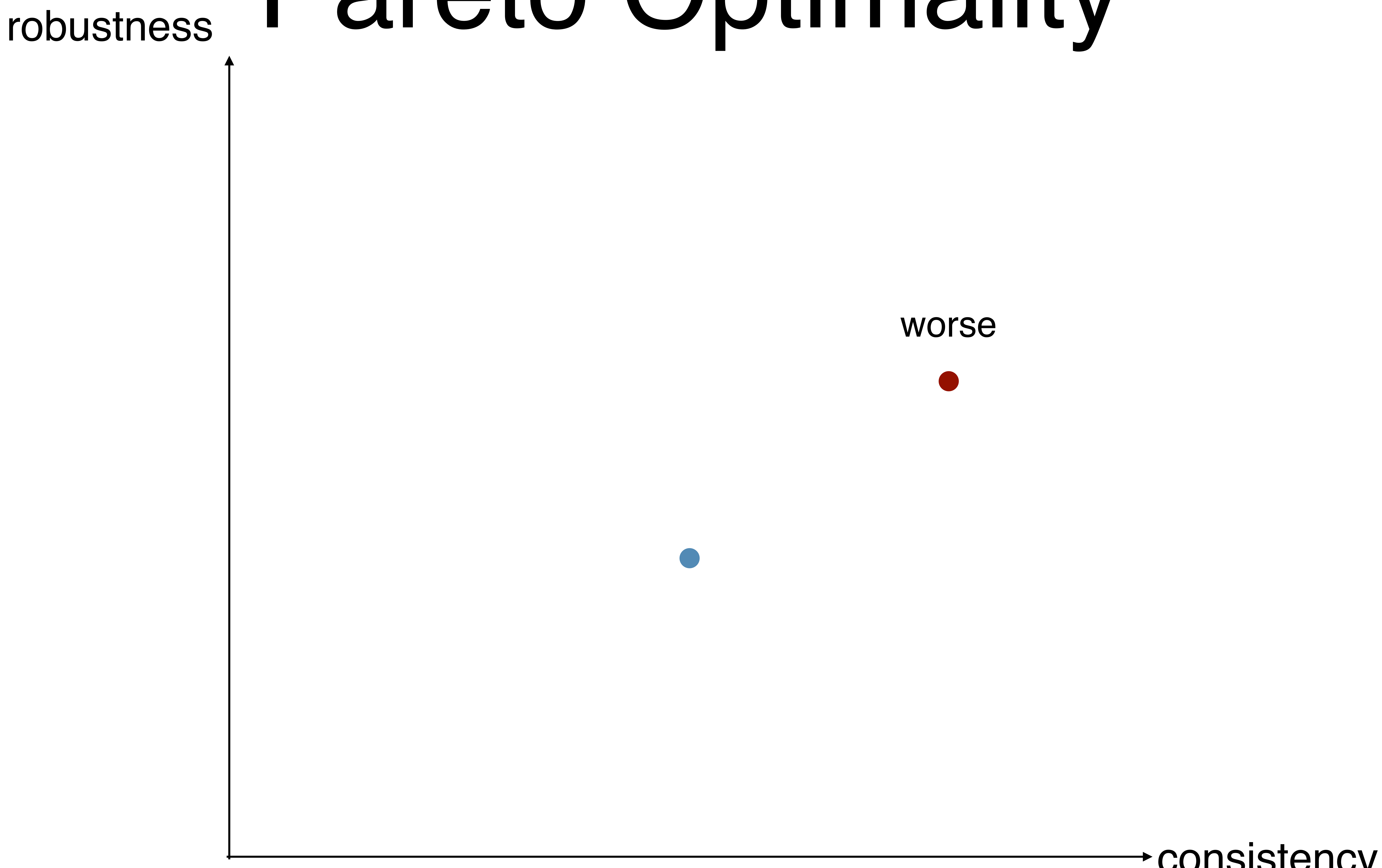

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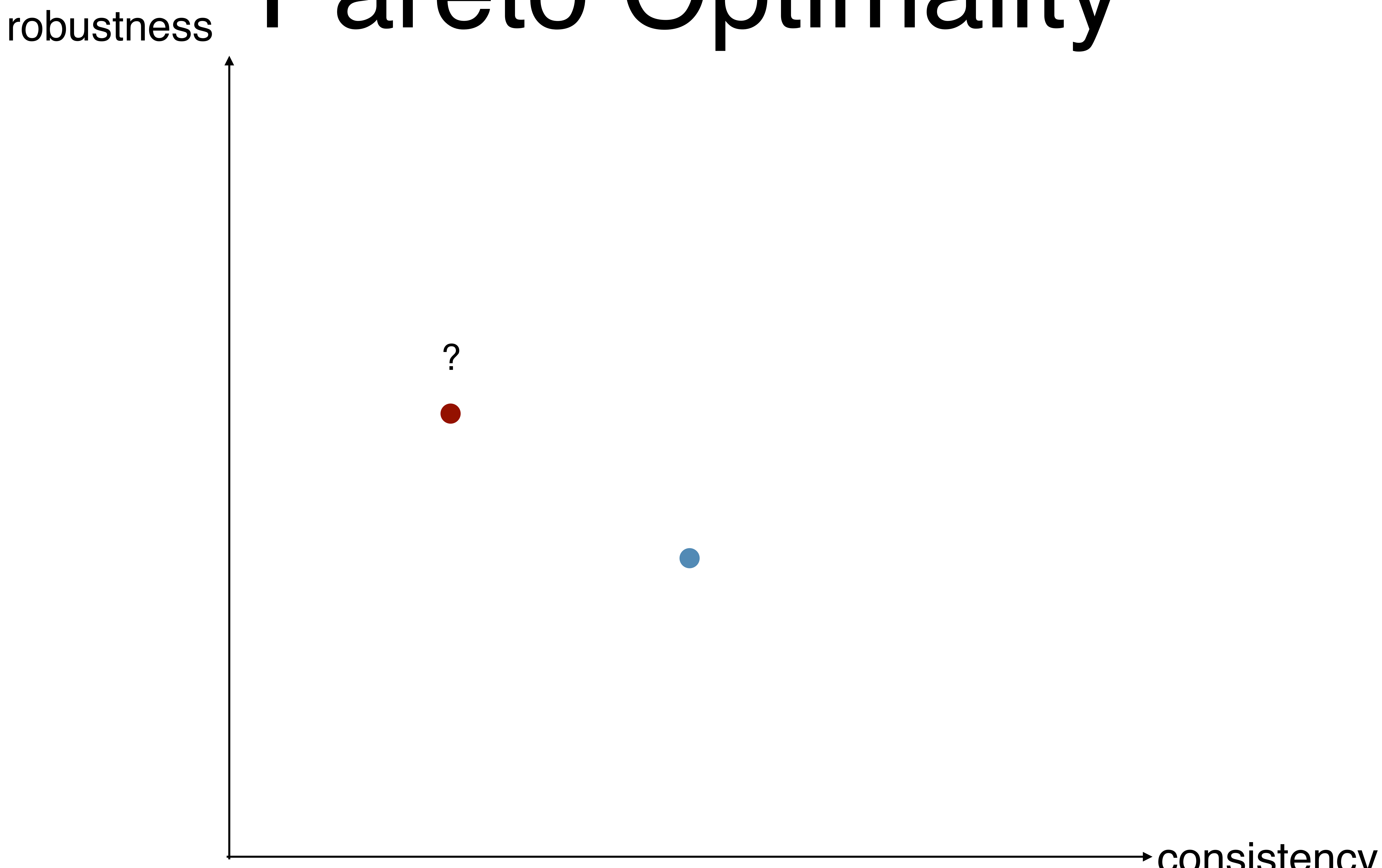

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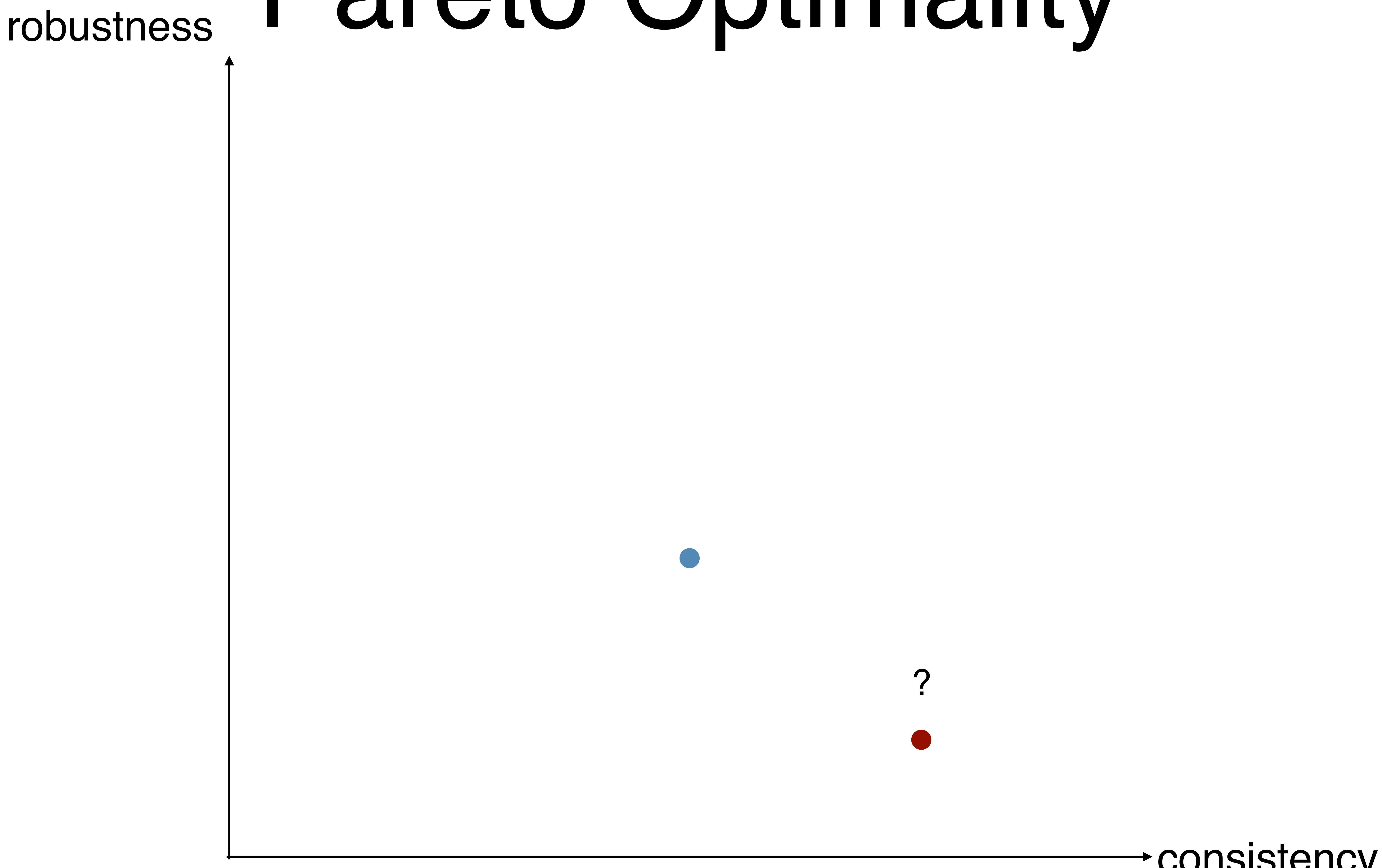


Pareto Optimality

What Happened

- In a bi-criteria optimization problem (minimizing criteria α and β),
  - ALG1 is better than ALG2 if $\alpha_1 \leq \alpha_2$ **and** $\beta_1 \leq \beta_2$
  - ALG is **Pareto optimal** if for all ALG', $\alpha \leq \alpha'$ **or** $\beta \leq \beta'$

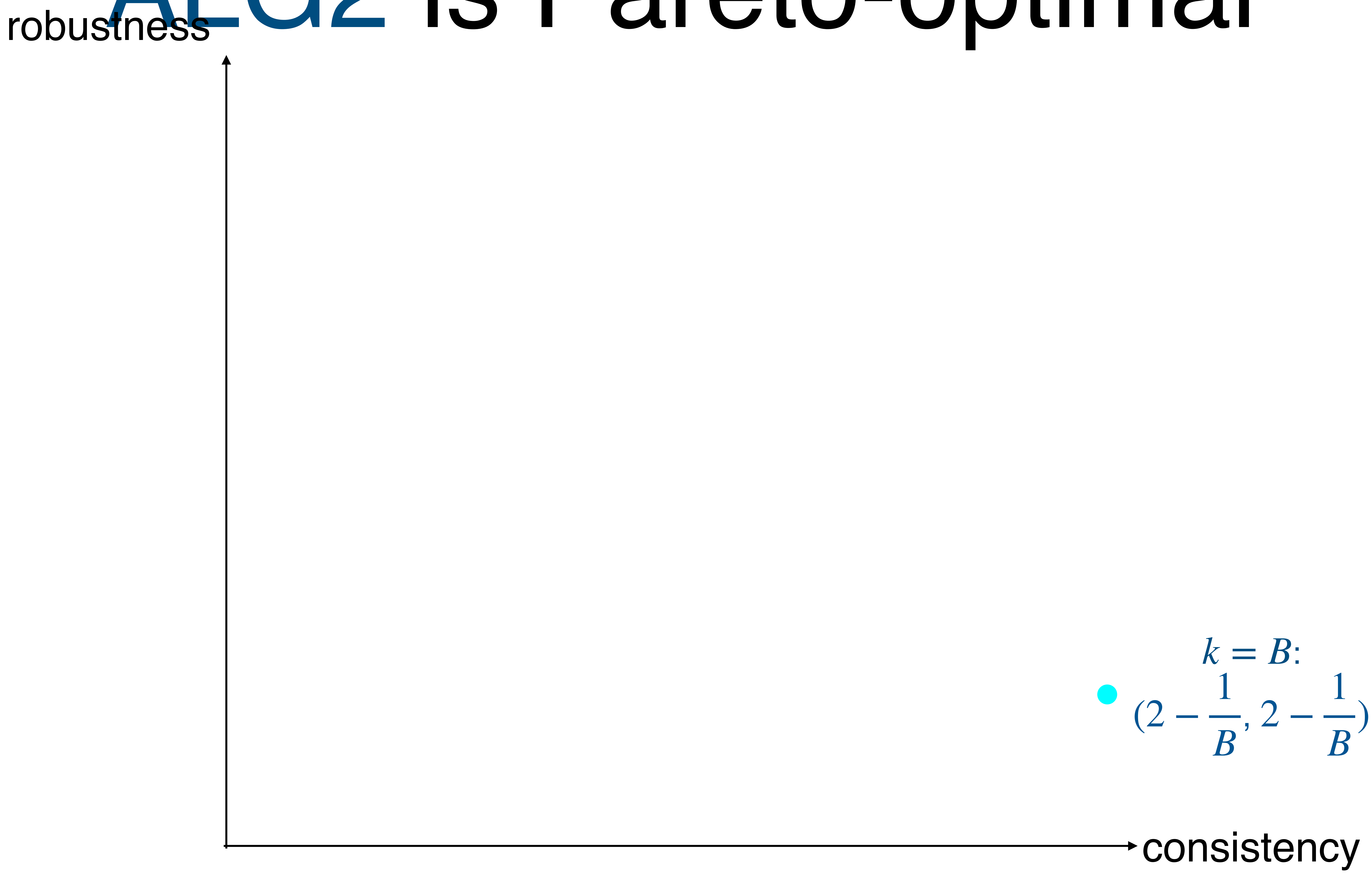
Ski Rental Algorithm 2

SKI-Rental with prediction (p, k)
  If $p \geq B$
    Keep renting until the k-th day // k is our "trust parameter"
  else ($p < B$)
    Keep renting until the B-th day

d: Actual number of skiing days

	Truth: $d \geq B$ (OPT buy)	Truth: $d < B$ (OPT rent)
Good prediction Consistency	Advice: $p \geq B$ $\frac{(k-1)+B}{B} = 1 + \frac{k-1}{B}$	Advice: $p < B$ $\frac{d}{d} = 1$
Bad prediction Robustness	Advice: $p < B$ $\frac{(B-1)+B}{B} = 2 + \frac{1}{B}$	Advice: $p \geq B$ $\frac{(k-1)+B}{k} = 1 + \frac{B-1}{k}$

ALG2 is Pareto-optimal



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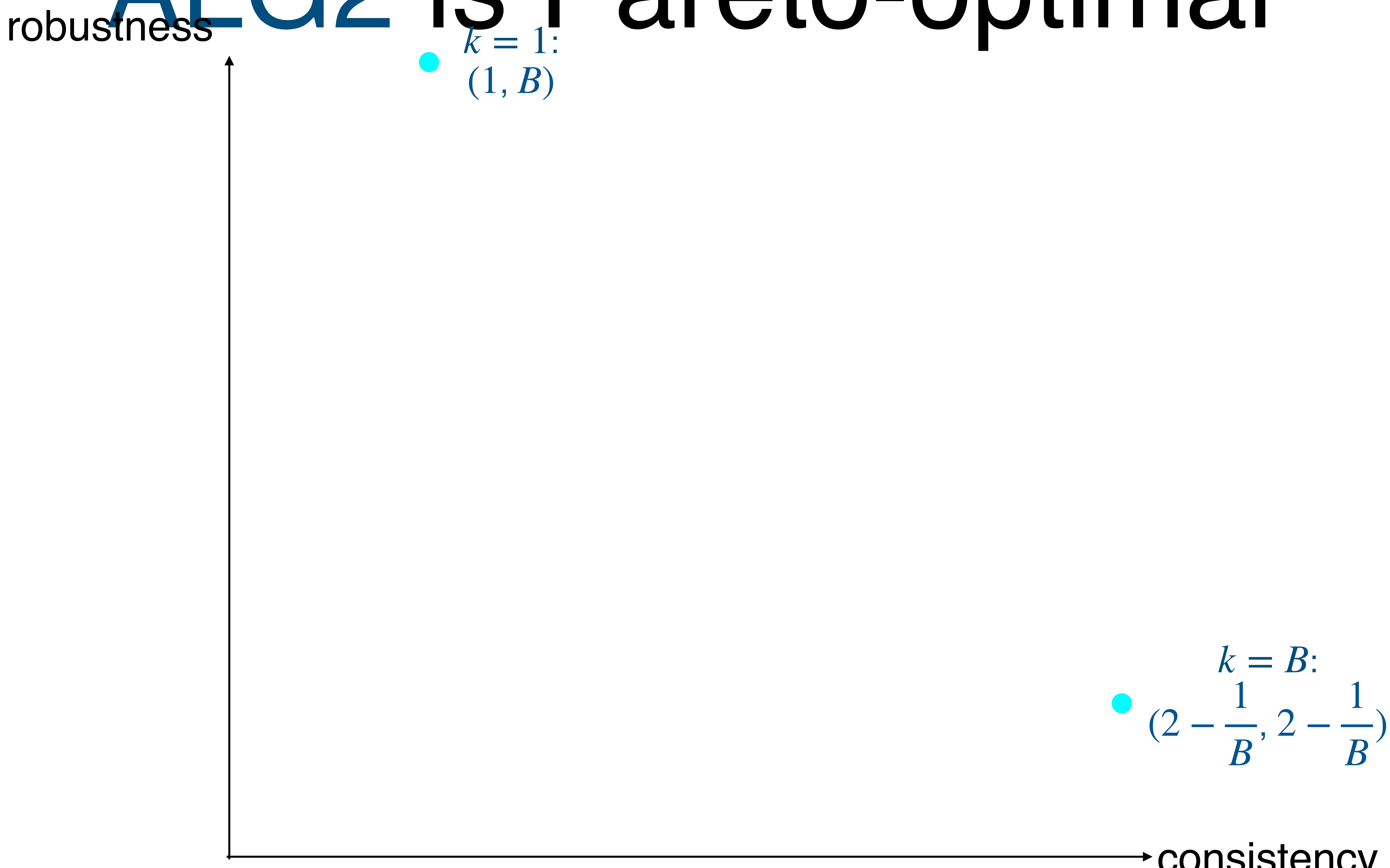

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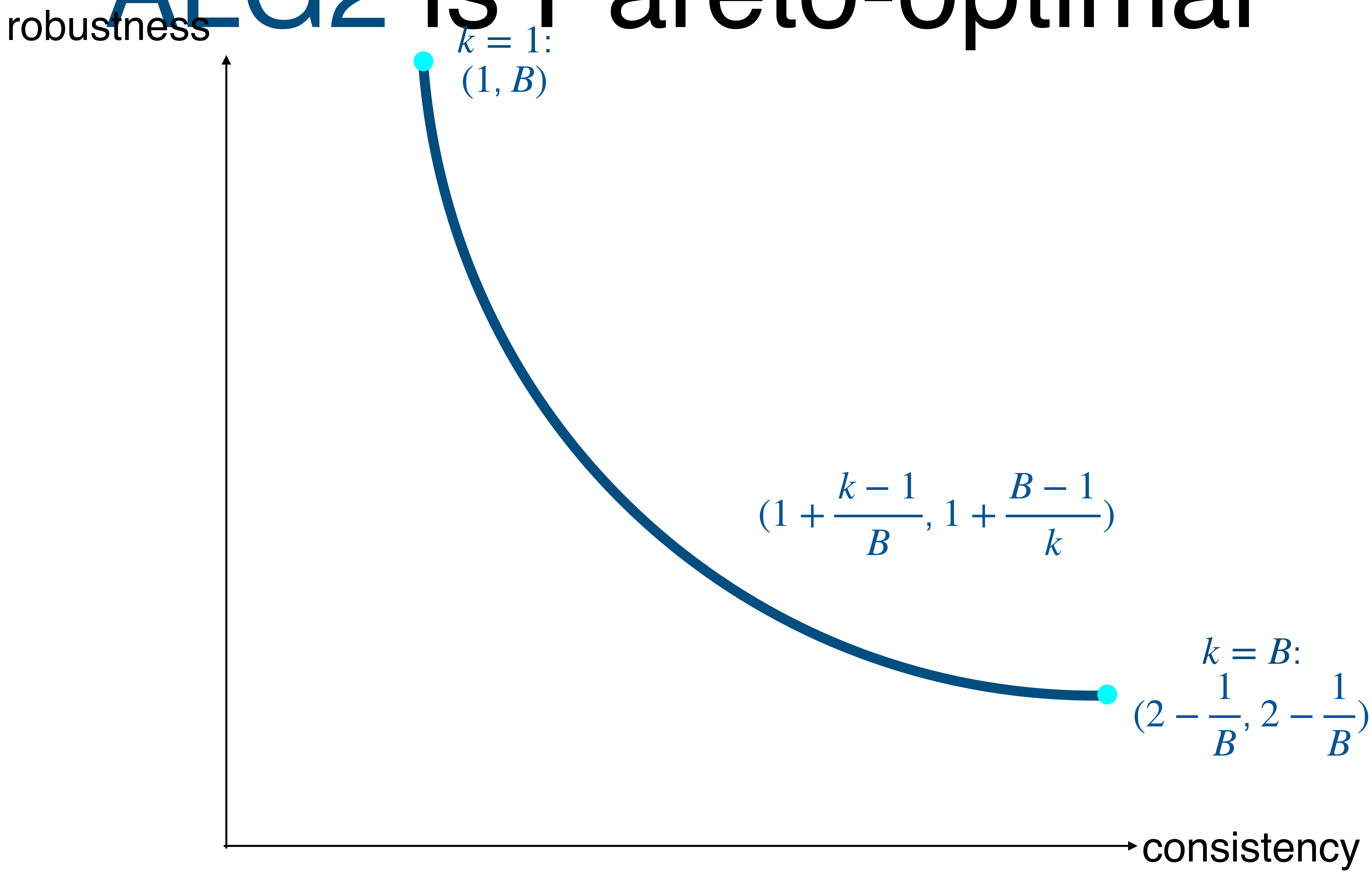

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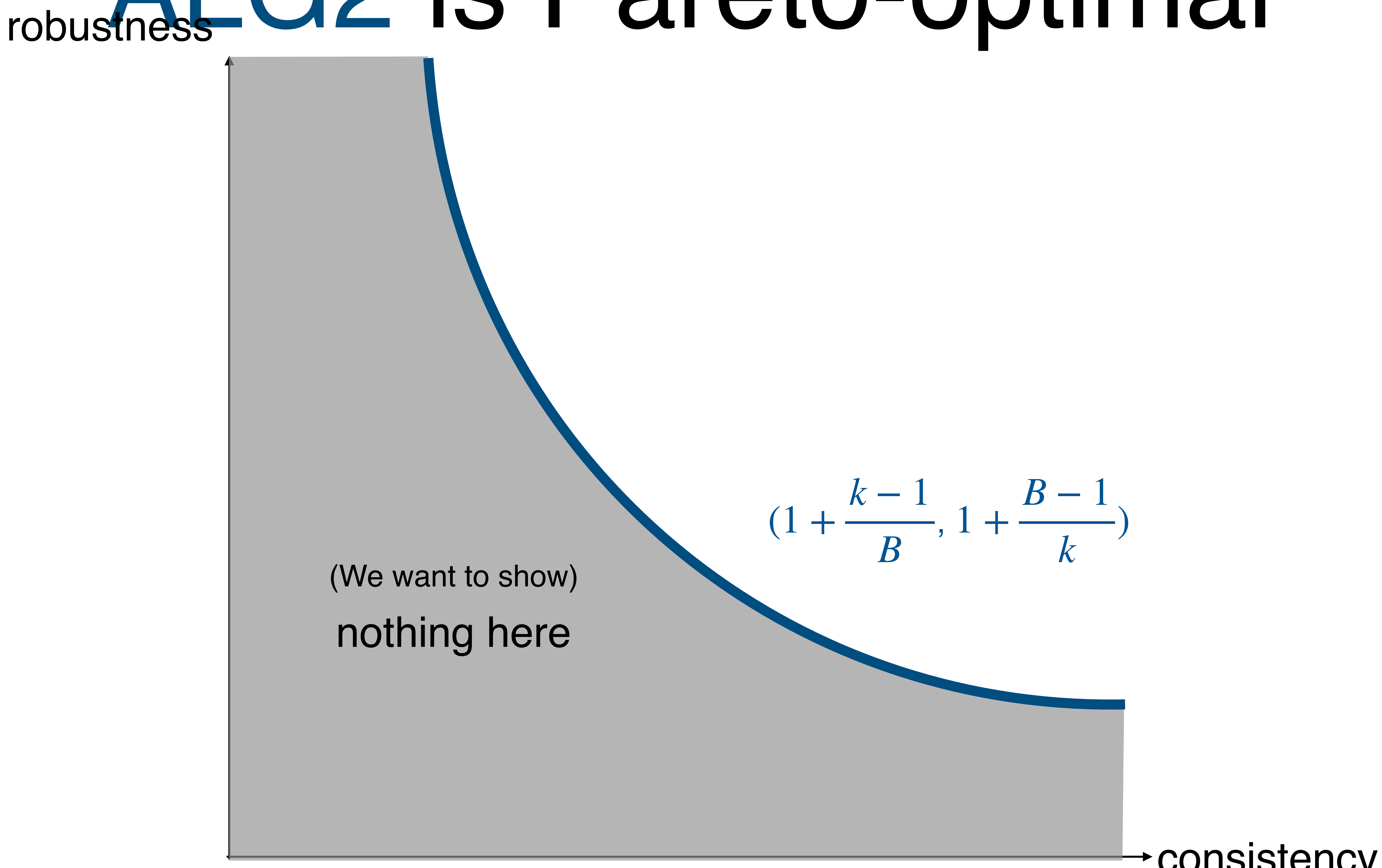
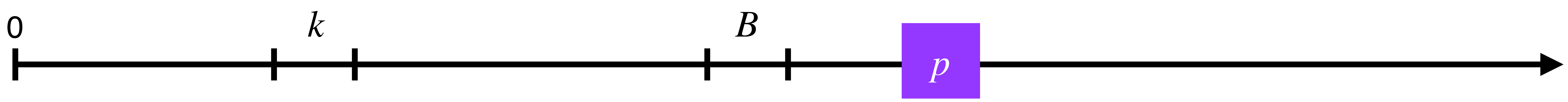


ALG2 is Pareto-optimal

Any algorithm A with consistency of at most $1 + \frac{k-1}{B}$ against prediction $p \geq B$ must buy the ski before the k-th day



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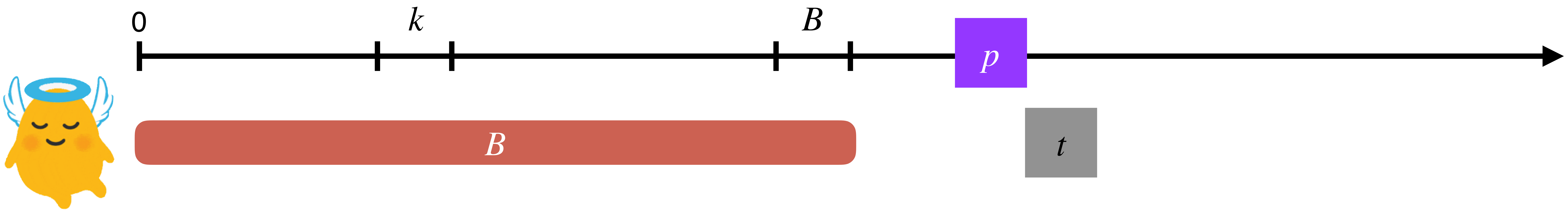
$\text{OPT} = B$



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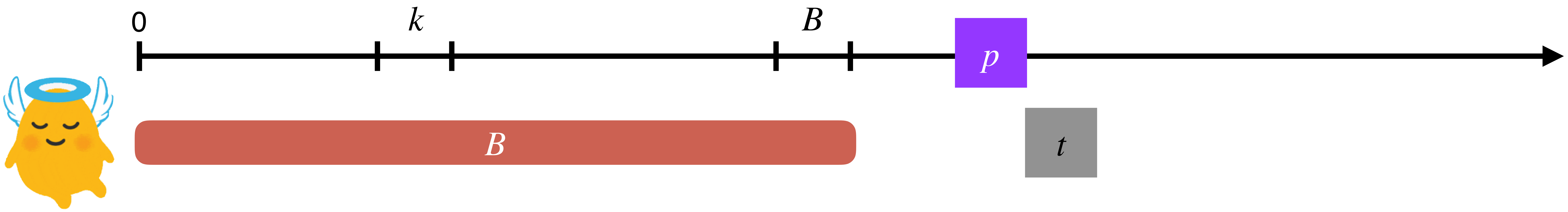

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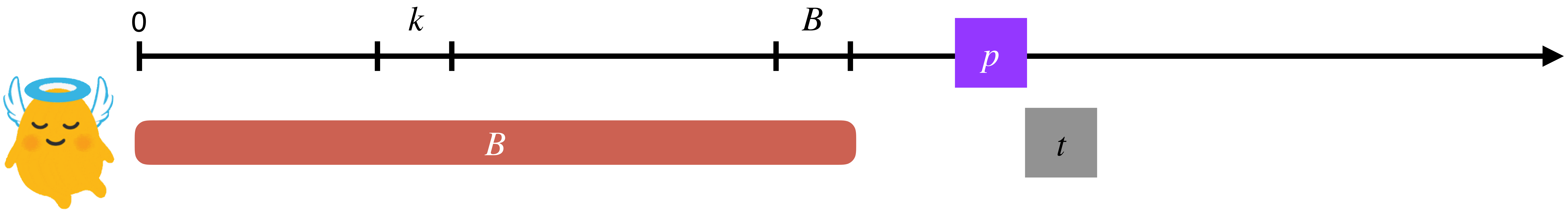
$\text{OPT} = B$

$\text{ALG} \leq (1 + \frac{k-1}{B}) \cdot B = B + k - 1$



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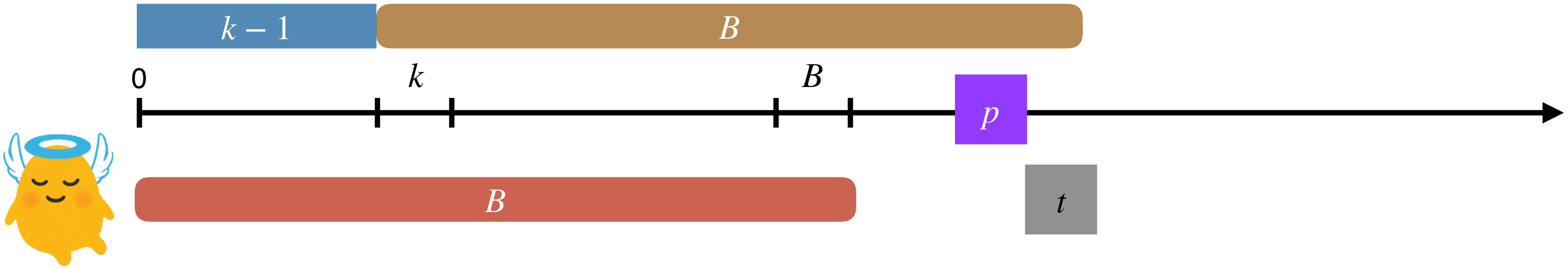
$\Rightarrow$ A $\left(1 + \frac{k-1}{B}\right)$-consistent algorithm must buy before the d-th day where $d \leq k$



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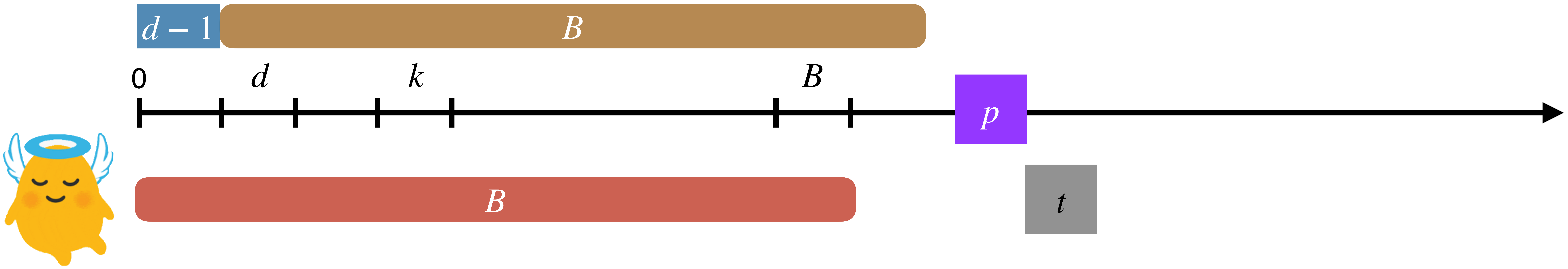

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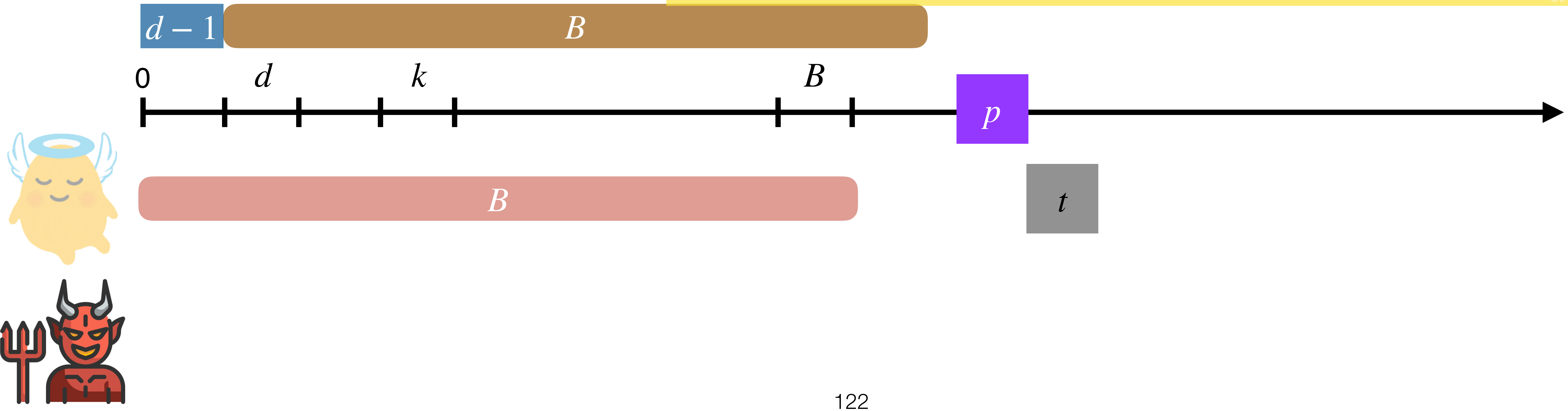
$\Rightarrow$ When the prediction is incorrect, OPT $= d$, and robustness $= \frac{d-1+B}{d} \geq 1$ -



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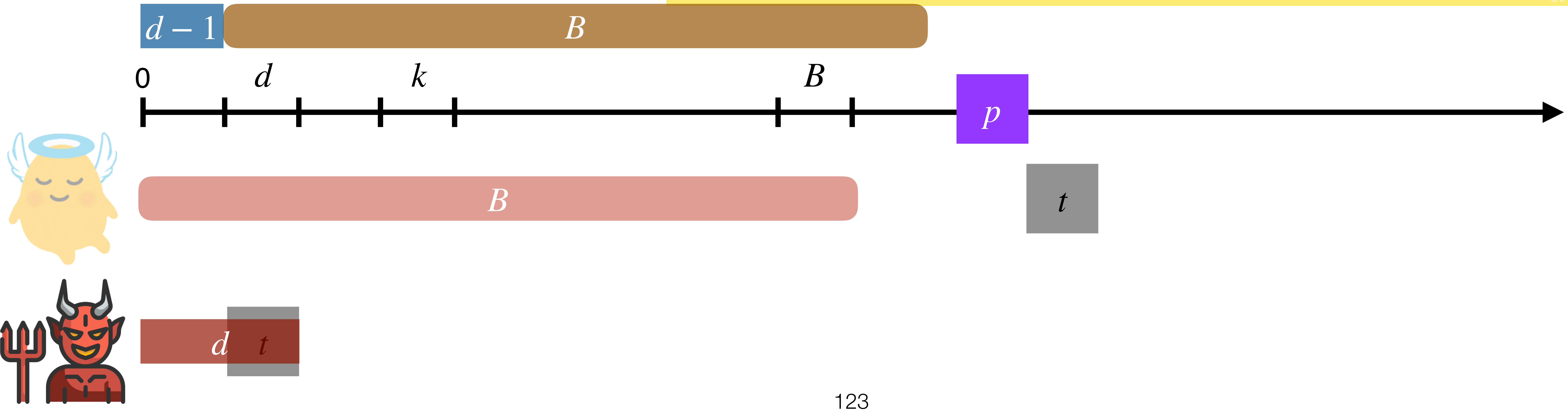

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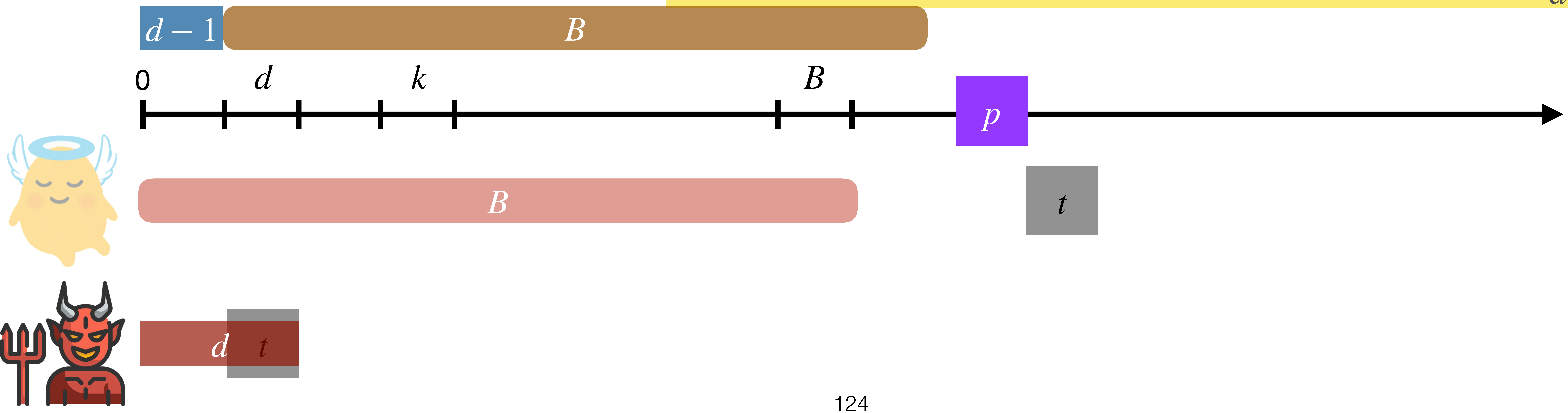

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Any algorithm A with consistency of at most $1 + \dfrac{k-1}{B}$ against prediction $p \geq B$ must buy the ski before the k-th day

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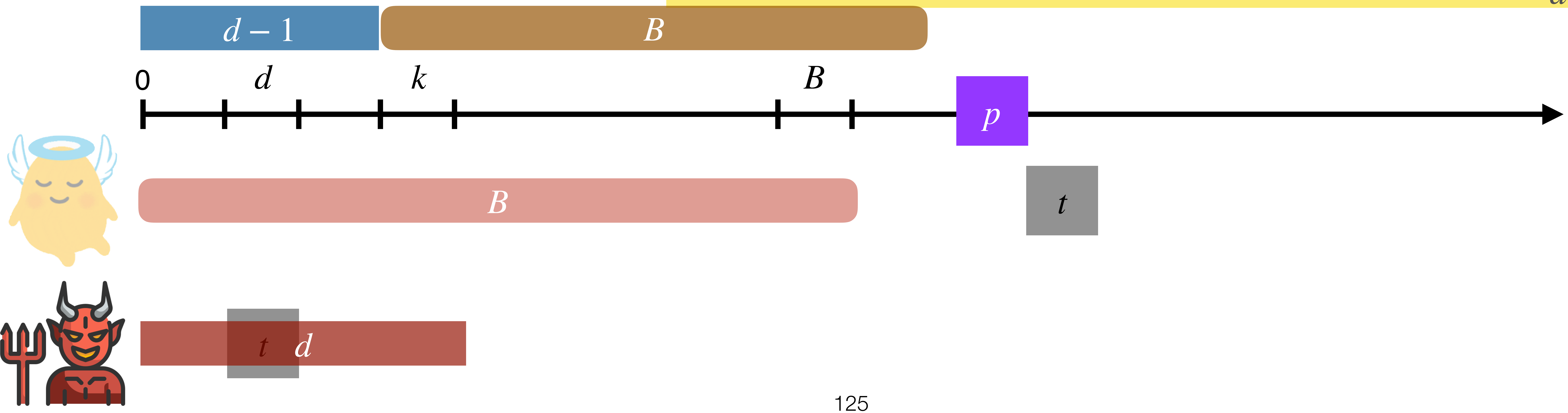
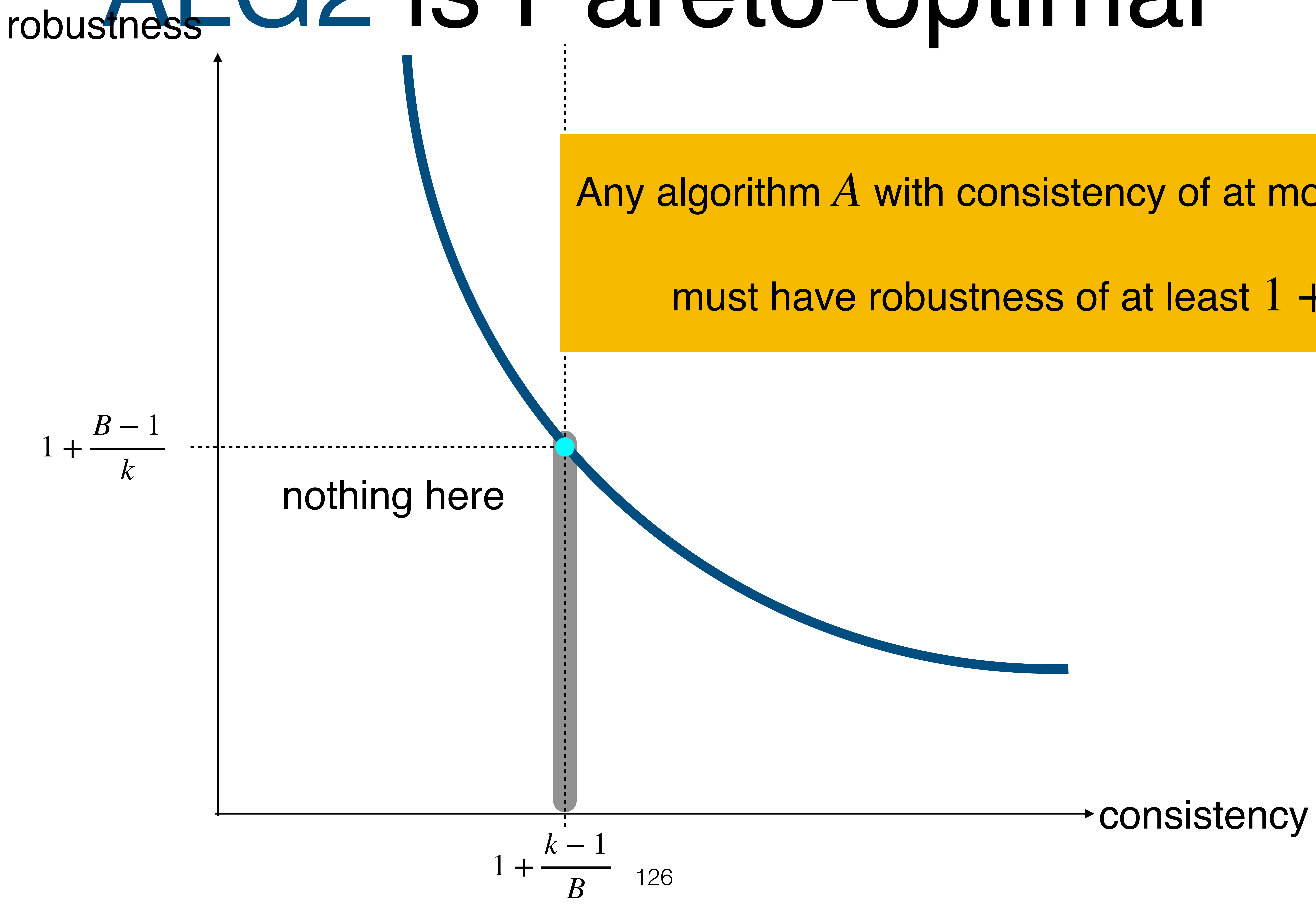


ALG2 is Pareto-optimal



Any algorithm A with consistency of at most $1 + \frac{k-1}{B}$ must have robustness of at least $1 + \frac{B-1}{k}$

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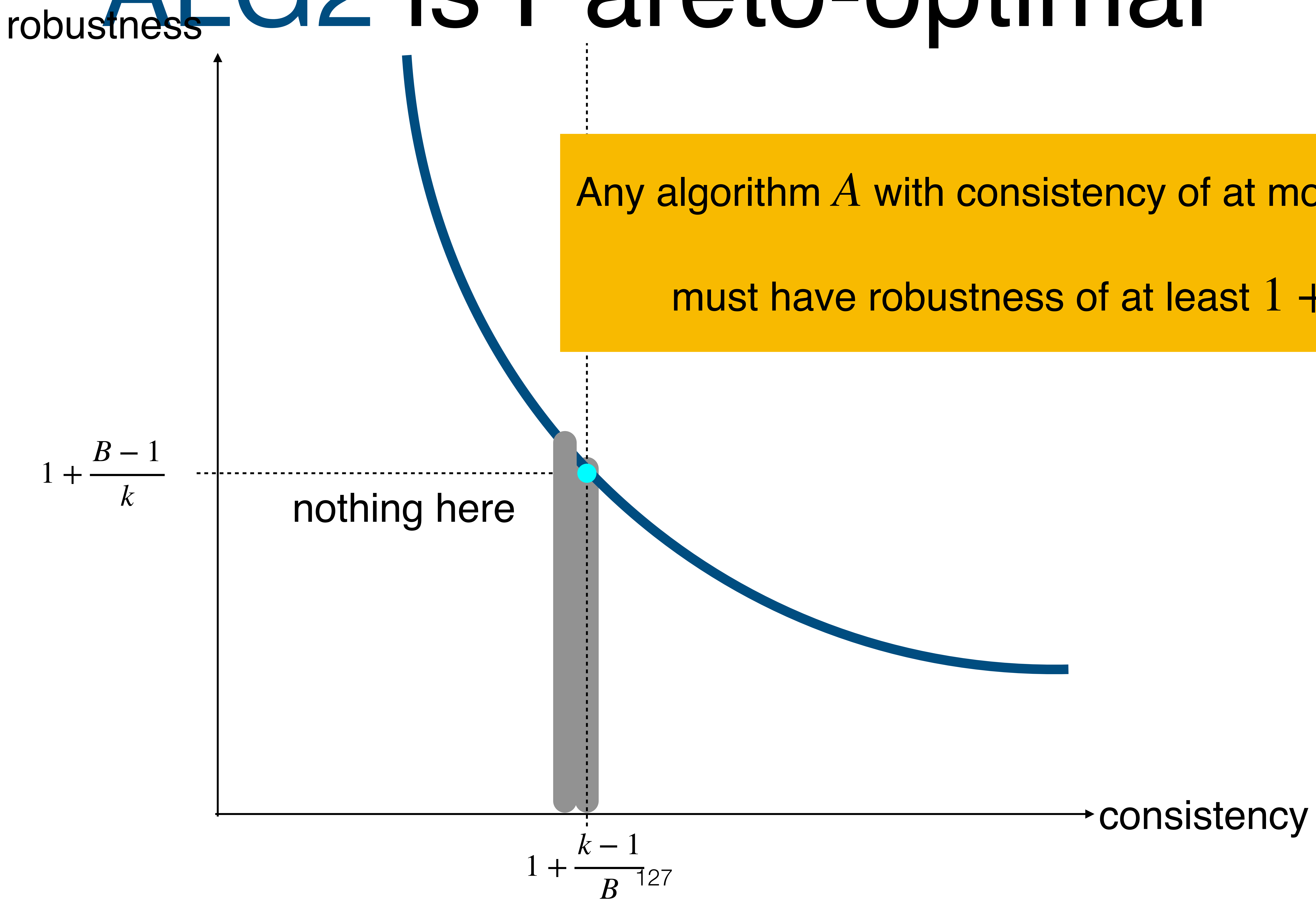

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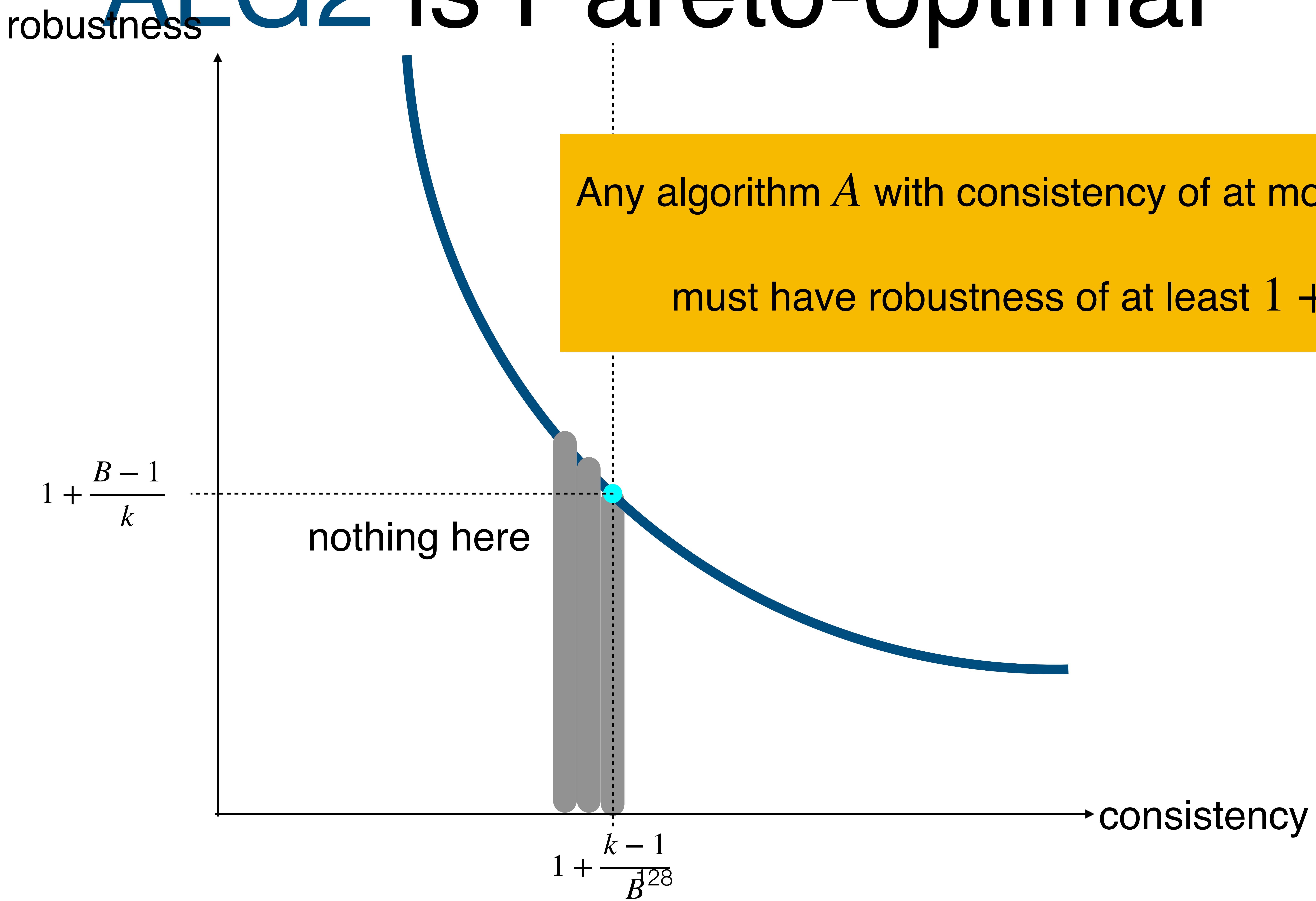

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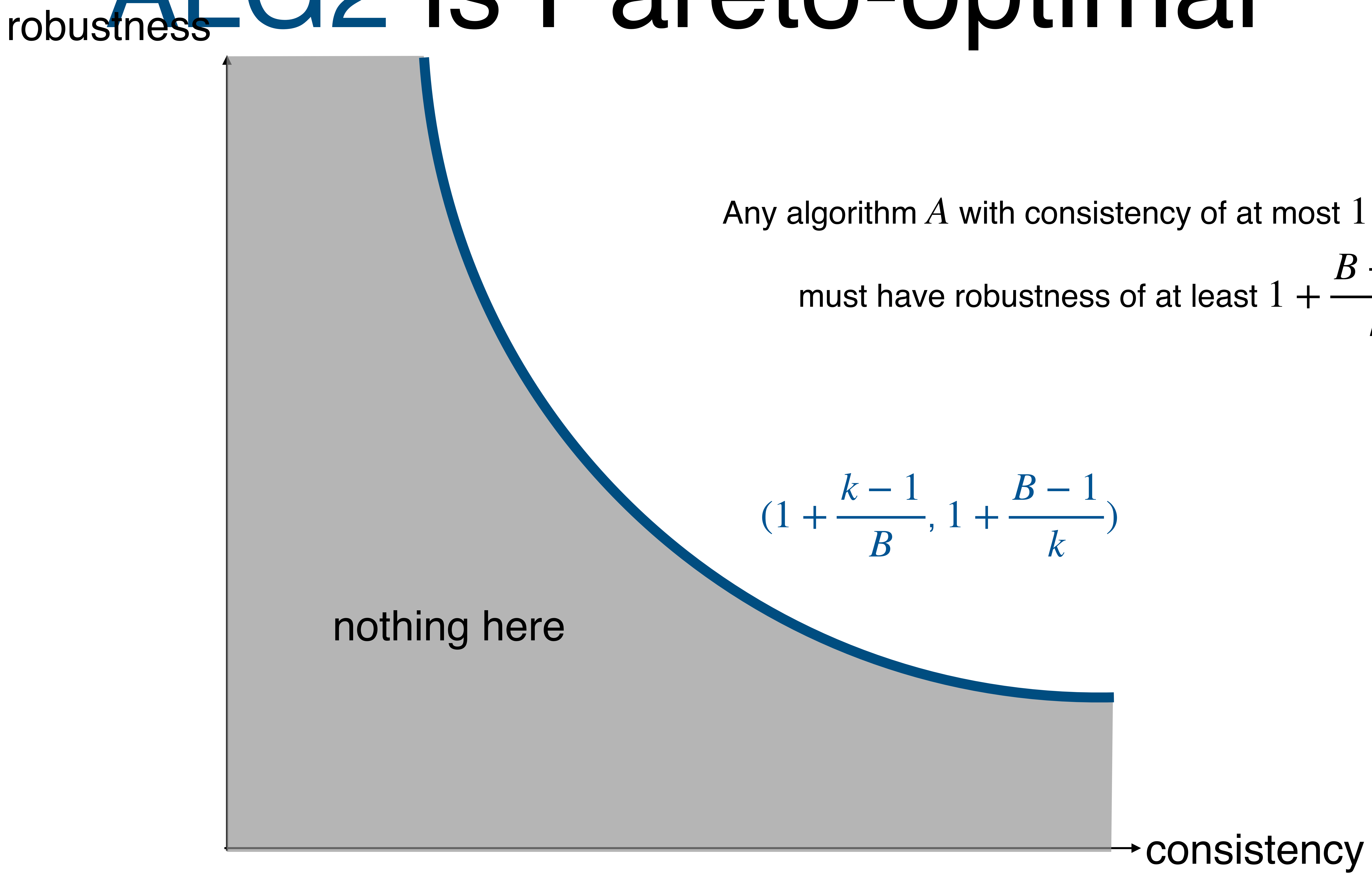

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Pareto Optimality