Exercise 1: Ever Given

1 Ever Given

In March 2021, a container ship *Ever Given* blocked the Suez Canal. Consider you are the captain of one of the ships that is waiting to pass through Suez Canal. You want to arrive at your destination as soon as possible. However, because of the obstruction, your ship is idle near the Suez Canal, and you have no idea when the canal will be available again. You have two options:

- to wait until the canal is available, and you can pass through the canal in F unit of time; or
- to go around Africa via the Cape of Good Hope, and you can arrive your destination in S units of time.

Note that F stands for fast, S stands for slow, and $F < S$.

Let p be the prediction about the time when the canal is available. Answer the following questions:

1. Consider the following algorithm. Analyze the consistency and robustness.

Algorithm 1 Algorithm 1 (p)

if $p \geq S - F$ **then**
  `Take the long path at time` 0
else $\triangleright\ p < S - F$
  `Wait until time` $S - F$ `(and then take the long path at time` $S - F$`)`

2. Let $k \in [0, S - F]$ be a trustness parameter. Consider the following algorithm. Analyze the consistency and robustness.

Algorithm 2 Algorithm 2 (p)

if $p \geq S - F$ **then**
  `Wait until time` k `(and then take the long path at time` k`)`
else $\triangleright\ p < S - F$
  `Wait until time` $S - F$ `(and then take the long path at time` $S - F$`)`

3. Follow Question 2. Show that the algorithm is Pareto optimal.

4. Let $\alpha \in [0, 1]$ be a trustness parameter. Consider the following algorithm. Show that the consistency of the algorithm is $2 - \alpha$ and the robustness of the algorithm is $\frac{2-\alpha}{1-\alpha}$.

Algorithm 3 Algorithm 3 (p)

if $p \geq S - F$ **then**
  `Wait until time` $(S-F)\cdot(1-\alpha)$ `(and then take the long path at time` $(S-F)\cdot(1-\alpha)$`)`
else $\triangleright\ p < S - F$
  `Wait until time` $\frac{S-F}{1-\alpha}$ `(and then take the long path at time` $\frac{S-F}{1-\alpha}$`)`

5. Follow Question 4. Let the error measure $\eta = |p - t|$, where p and t are the predicted and the actual time when the canal is available, respectively. Analyze Algorithm 3 and show that the consistency is $2 - \alpha$ and the robustness is $\max\{1 + \frac{\eta}{\alpha \cdot \mathrm{OPT}}, 2 - \alpha + \frac{(2-\alpha)\cdot\eta}{\mathrm{OPT}}\}$, where OPT is the optimal cost.