Exercises - Divide and Conquer (2) and RP - Algoritmiek

Tutorial February 8, 2024

1. **From A to B:** Suppose we are given integers a, b such that $2 \leq a \leq b$. Consider two operations: *Increment* (I), which increments a number by 1, and *Double* (D), which doubles a number. You are asked to find the minimum number of operations to get from a to b. For example, you can get from 8 to 38 in 4 steps (IDID).

   (a) Determine the list of choices and top choice for this problem.

   (b) Formulate a clear subproblem.

   (c) Give a recurrence for your top choice and subproblem.

   (d) Prove the optimality principle for your top choice and subproblem.

2. **Exact change: alternative method:** For the Exact Change problem, we can also consider the following list of choices: how many coins of value a_1 do we return, how many coins of value a_2, etc.

   (a) Define the top-choice and the subproblem you need to use. Does one parameter for your subproblem suffice?

   (b) Give the recurrence for this formulation of the problem. Don't forget the base case!

   (c) Prove that the optimality principle holds.

   (d) Discuss the relation of this recurrence with the recurrence discussed in the lecture.

3. **Windmills:** Windmills are being built all along the Dutch coast. There are n possible locations to build windmills, say on a line, where location i lies 1km to the north of location $i-1$. Due to wind conditions, by building a windmill at location i you harvest energy e_i. To not disrupt nature and windflow, windmills must be built at least K kilometers apart, for some integer $K \geq 1$. Subject to this condition, what is the maximum amount of wind energy you can harvest by appropriately placing windmills?

   (a) Determine the sequences of choices made and define the top-choice and subproblem.

   (b) Give a recurrence following your subproblem and top-choice. Don't forget the base case!

   (c) Prove that the optimality principle holds.

4. **Dominatie in lineaire tijd:** Consider the domination ("Beste Koop") problem from the lecture. Give an $O(n)$ time, divide & conquer algorithm for this problem. You may assume the input is already sorted on quality. Give the algorithm, the recurrence for its running time, and prove that the recurrence is bounded by $O(n)$.

5. **Mister Animal:** Mister Animal has a video channel and in his new video, he has the following challenge for an unsuspecting viewer, you! Mister Animal gives you B dollars to shop equipment in an electronics store. The store has n item of value $v_1, \ldots, v_n$; each item can be found only once in the shop. The challenge is to spend exactly B dollars; if you fail, you do not get to keep the electronics and instead get pelted with water balloons. Before you start your spending spree, you strategize and look for a solution that spends exactly b dollars.

   (a) Determine the list of choices and the top choice for this problem. Think again about the number of parameters you might need.

   (b) Formulate a clear subproblem.

   (c) Give a recurrence for your top choice and subproblem.

   (d) Prove the optimality principle for your top choice and subproblem.