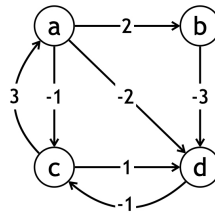


Exercises - All Pairs Shortest Paths - Algorithms Tutorial

1. **Practice:** Run Johnson's algorithm on this example:



2. **Graphs with negative cycles:** Suppose you are given a directed graph G which may have negative cycles. Describe and analyze an $O(|V||E|)$ time algorithm that computes the shortest-path distances from s to every other vertex in G . Specifically, for every vertex v :
- If v is not reachable from s , your algorithm should report $d[v] = \infty$
 - If G has a negative cycle that is reachable from s , and v is reachable from that cycle, then $d[v] = -\infty$
 - If neither of the two previous conditions applies, your algorithm should report the correct shortest-path distance from s to v .
3. **Arbitrage cycle:** The arbitrage business is a money-making scheme that takes advantage of differences in currency exchange. In particular, suppose 1 US dollar buys 120 Japanese yen, 1 yen buys 0.01 euros, and 1 euro buys 1.2 US dollars. Then, a trader starting with 1 dollar can convert their money from dollars to yen, then from yen to euros, and finally from euros back to dollars, ending with 1.44 dollars. The cycle of currencies dollars $\rightarrow$ yen $\rightarrow$ euros $\rightarrow$ dollars is called an arbitrage cycle. Of course, finding and exploiting arbitrage cycles before the prices are corrected requires extremely fast algorithms. Model the problem as a graph algorithm problem.
4. Show that there is a negative cycle if and only if, at the end of FLOYD-WARSHALL algorithm, $dist_{vv}^{(n)} < 0$ for some v .